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Review Article

On symmetries, resonances and photonic crystals in morphogenesis

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ABSTRACT

Biological symmetries, theories of the morphogenetic field, resonant interactions and the role of photons in morphogenetic processes represented the main fields of interest of Lev Beloussov and his followers. This review article includes some results of our study on the important role of resonances and photonic crystals in genetic informatics. Mathematical formalisms of differential Riemannian geometry and tensor analysis are used for modeling inherited curved surfaces in biomorphology and for understanding conformal bio-symmetries connected with the networks of curvature lines of surfaces. Notions of a morpho-resonance field as one of variants of morphogenetic fields are discussed. The connection of the golden section with the Fibonacci matrix of growth used in morphogenetic models of phyllotaxis is shown. Photonic crystals are considered as important participants of organisation of molecular-genetic informatics.

1. Introduction

In studying the problems of morphogenesis, the phenomenon of symmetries in biological bodies is traditionally assigned a prominent place. Scientific interest in biosymmetries is based on the especially important role of the concept of symmetry and the group-theoretical approach in modern mathematical natural science. Besides all, "*symmetry in the broad or narrow sense is the idea using which man has for centuries tried to obtain an insight into and create order, beauty and perfection*" (Weyl, 1952). Biological symmetry is embodies to a larger or smaller degree in numerous biological theories, some highly controversial: N.I. Vavilov's law of homological series; A.G. Gurwitsch's theory of the morphogenetic field (Beloussov, 1997); V.I. Vernadsky's theory of the non-Euclidean geometry of living matter; the diffusion reaction model of morphogenesis, developed by A.M. Turing; self-organizing growing automata, whose theory is being developed by J. von Neumann's followers; morphogenetic mechanisms behind numerous psychophysical phenomena including the esthetic preference of the morphogenetically significant golden section, which is expressed by Fibonacci numbers etc.

The morphological variability follows certain rules that can be called nomothetical laws and analyzed as symmetrical transformations (Meyen, 1973). The nomothetical laws and morphogenetic phenomena are related with the known idea about existence of a morphogenetic field, a possible nature and bases of which are discussed by many authors (Beloussov, 1998, 2012, 2015; Igamberdiev, 2014; Meyen, 1973). The article (Levine, 2011) describes a modern understanding of morphogenetic fields as information-bearing global patterns in chemicoelectrical properties that guide growth and form; it is «*a profound unifying concept central to biology and medicine*».

This issue is dedicated to the memory of the remarkable embryologist Lev V. Beloussov, many of whose researches were devoted to the systematic analysis of symmetries in embryological structures and theories. His fundamental books (Beloussov, 1998, 2015) contain extensive material on this topic. In many of his works L.Beloussov continued study of morphogenetic phenomena in close connection with approaches and the concept of the morphogenetic field developed by famous Russian physiologist A.G. Gurwitsch, who was his grandfather. Beloussov has organised special international conferences to develop scientific achievements by Gurwitsch concerning, first of all, his concept of the morphogenetic field, resonant coherent biosystems and also the important role of photons in morphogenetic phenomena (Beloussov et al., 2007). The same issues were intensively discussed at seminars on problems of morphogenesis, conducted by L. Beloussov for many years in Moscow. One of the authors of this article - Petoukhov - was lucky to be a regular participant in these seminars, which had a significant influence on him.

Our article continues the theme of biological symmetries and mechanisms of morphogenesis in connection with the symmetric features of the genetic coding system. Developing and supplementing some ideas of L. Beloussov and A. Gurwitsch, this article pays a special attention to the role of resonant interactions and photonic crystals in the

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development of model approaches to understanding inherited morphogenetic structures. From the information standpoint, biological organisms are informational essences. They receive genetic information from their ancestors and transmit it to descendants. Science has discovered that all organisms are identical to each other by their basic molecular-genetic structures. Due to this revolutionary discovery, a great unification of all biological organisms has happened in science. A new understanding of life itself has appeared: "*life is a partnership between genes and mathematics*" (Stewart, 1999).

Mathematical analysis of symmetrical structures and phenomena of genetic systems have revealed their connections with matrix theory of resonances in oscillatory systems having two and more degrees of freedom. This had led to the concept of the resonance genetics and to the idea of morphoresonance field (Petoukhov, 2015a, 2016). This article gives additional data to develop modeling approaches for understanding inherited morphogenetic phenomena taking into account the works of Beloussov and Gurwitsch.

Informational genetic molecules DNA and RNA exist on principles of quantum mechanics and quantum informatics, but they encode structures of living macro-organisms, which are subjects of classical mechanics. Mathematics of resonances of oscillatory systems is appropriate for quantum mechanics and classic mechanics since such mathematics uses in both cases the same property of matrices to express resonances. Morphogenesis is based on addition or subtraction of new molecules in existing constructions by special interactions among molecules. Let us remind some known facts and data from physics about the key role of resonances and photons in such molecular interactions.

2. On mathematics of resonances and their applications

The concept of resonances plays fundamental and interdisciplinary role in science. In classic mechanics, the concept of resonances has wide theoretical and engineering applications due to vibrational phenomena of a resonant synchronization of oscillatory processes, vibrational separation and structuring of multiphase systems, vibro-transportation of substances, vibro-transmission of energy within systems, *etc*. (Blekhman, 2000; Ganiev et al., 2015). Practically invisible vibrations can provide, for example, the following phenomena: the upper position of the inverted pendulum becomes stable; heavy metal ball “floats” in a layer of sand; a rope takes a form of a vertical stem if a corresponding vibration acts on its base. Inside fluids, vibrating bodies can attract or repel each other (vibrating forces of Bjerknes) and pulsating gas bubbles may coalesce or divide.

Quantum mechanics has begun in 1900 due to works by M. Planck, who has analyzed a great set of resonant oscillators inside the cavity and in the result has received his famous law of electromagnetic radiation emitted by a black body in thermal equilibrium. Later, after more than 50 years of successful development of quantum mechanics, E. Schrodinger emphasised the basic meaning of resonances: “*The one thing which one has to accept and which is the inalienable consequence of the wave-equation as it is used in every problem, under the most various forms, is this: that the interaction between two microscopic physical systems is controlled by a peculiar law of resonance*» (Schrödinger, 1952, p.115). In considering an exact balance in nature between bundles of energy, lost by one system and gained by another, he noted: «*I maintain that it can in all cases be understood as a resonance phenomenon*» (Schrödinger, 1952, p.114). He wrote in his resonance concept of quantum interactions, that chemical reactions, including photochemical reactions, can be explained on the base of resonances. One of examples considered in his article was a production of water molecules H_2O from a suitable mixture of hydrogen gas H_2 and oxygen gas O_2 under action of ultraviolet light. In this example, “*wave-mechanically the gaseous mixture is represented by a vibration of the combined system, and, by the way, not by one proper vibration since there is anyhow the vast variety of translational and rotational modes, and, of course, the electronic modes. The gaseous compound, H_2O, is represented by an entirely different vibration of the same system*” (Schrödinger, 1952, p. 118).

His book (Schrödinger, 1944) said that the chromosome is an aperiodic crystal since its atoms are connected each other by forces of the same nature that atoms in crystals. But vibrations and resonances play a very important role in physics of crystals and their morphological structure. The interaction of atoms in the crystal lattice together with the resonance phenomenon leads to the fact that oscillatory motions of lattice elements are combined in a collective oscillation process in a form of a wave propagating in the crystal. In the course of the normal vibrations, all the atoms in the crystal lattice oscillate about their equilibrium positions by harmonic law with the same frequency. As it is known, in a quantum description of small oscillations of a crystal, it is possible to interpreted normal fluctuations of the crystal as special quasiparticles, which are quanta of the field of elastic vibrations of the crystal and which are called phonons. The theory of phonons is one of the bases of physics of crystals. We believe that a similar resonance approach can usefully serve in morphogenetics.

L.Pauling used ideas of resonances in quantum mechanical systems in his theory of resonance in structural chemistry. His book (Pauling, 1940) about this theory is the most quoted among scientific books of the 20th century. The theory was developed to explain the formation of hybrid bonds in molecules. The actual molecule, as Pauling proposed, is a sort of hybrid, a structure that resonates between the two alternative extremes; and whenever there is a resonance between the two forms, the structure is stabilized. His theory uses the fundamental principle of a minimal energy because – in resonant combining of parts into a single unit – each of members of the ensemble requires less energy for performing own work than when working individually. Of course, this fundamental principle can be used in many other cases of resonances in different systems as the physical base. The principle of energetic minimum in resonance processes has some correlations with the principle of relaxation in morphogenetic processes proposed in (Igamberdiev, 2012). From the point of view of quantum mechanics, the interaction of molecules is based on the emission and absorption of photons with the participation of resonance correspondences.

The notion “resonance” was introduced into quantum mechanics by W. Heisenberg in 1926 year in connection with analyzes of multi-body systems. He emphasized that in quantum mechanics the phenomenon of resonances has much more general character than in classical physics. In classic theory, two periodic oscillating systems come into their own resonance only in the case when a frequency of a separate sub-system doesn’t depend on energy of the system and when this frequency is approximately equal in both sub-systems. In quantum mechanics, two atomic systems come into their resonance only in the case when a frequency of absorption of one system coincides with a frequency of emitting another system, or vice versa (Heisenberg, 1926, §2). Quantized electromagnetic field is represented as a set of oscillators.

Further development of thoughts by Beloussov, Gurwitsch and other researches about important role of resonances in biological phenomena can be done on the basis of studing structural analogies between morphogenetic phenomena and mathematical formalisms of the theory of resonances. Mechanical and electrical oscillations in living bodies are closely connected because many tissues are piezo-electrical (nucleic acids, bone, actin, dentin, tendons, etc.). Mathematics of mechanical and electrical oscillations is analogical (so called “electro-mechanical analogies” are well-known). The articles (Petoukhov, 2015a, b, 2016; Petoukhov, Petukhova, 2017) describe our concept about the important role of resonances in genetic structures. This concept is based on impressive analogies of some genetic structures, including Mendelian laws, with eigenvalues and eigenvectors of tensor families of matrices representing resonant characteristics of oscillatory systems with many degrees of freedom (see about resonant characteristics in Gladwell (2004)). For example, concerning Mendelian laws, known for a long time Punnet squares for poly-hybrid crosses of organisms are identical to the tensor inheritance for spectra of vibrosystems with appropriate degrees of freedom (Petoukhov, 2016). The concept of resonance genetics draws attention to a possible value of phenomena of vibrational mechanics in physiology with its complex phenomena of coordinated actions of many parts, for example, within division of cells, etc. For further parts of this article we should briefly recall the main aspects of matrix representations of resonances in oscillatory systems.

Matrices possess a wonderful property to express resonances, which sometimes is called as their main quality (Bellman, 1960; Balonin, 2000, p. 21, 26). Physical resonance phenomenon is familiar to everyone. The expression y = A*S models the transmission of a signal S via an acoustic system A, represented by a relevant matrix A. If an input signal is a resonant tone, then the output signal will repeat it with a precision up to a scale factor y = A*S by analogy with a situation when a musical string sounds in unison with the neighboring vibrating string. In the case of a matrix A, its number of resonant tones S_i corresponds to its size. They are called its eigenvectors, and the scale factors λ_i with them are called its eigenvalues or, briefly, spectrum A (see more details in the special article in this journal Petoukhov (2016)). Each eigenvalue of the vibration system matrix is equal to the square of the corresponding resonant frequency. Frequencies $\omega_i = \lambda_i^{0.5}$ (Gladwell, 2004, p. 61) are defined as natural frequencies of the system, and the corresponding eigenvectors are defined as its own forms of oscillations (or simply, natural oscillations). These free undamped oscillations occur in the system in the absence of the friction forces in it and in the absence of external excitation forces. Behavior of the system in conditions of free oscillations determines by its behavior in many other conditions. In this context, one of the main tasks of the theory of oscillations is a determination of natural frequencies (mathematically, eigenvalues of operators) and the natural forms of oscillations of bodies. To find all the eigenvalues λ_i and eigenvectors of the matrix A, which are defined by the matrix equation $A^*s = \lambda^*s$, the “characteristic equation” of the matrix A is analyzed: $\det(A - \lambda E) = 0$, where E – the identity matrix. The characteristic equation together with its eigenvalues and eigenvectors is fundamental in the theory of mechanical, electrical and other oscillations at macroscopic or microscopic levels. Not all square matrices represent vibrational systems. Matrices, which are relevant to the various problems of the theory of oscillations, are usually symmetrical real matrices (Gladwell, 2004, p. 178). Such matrices have real eigenvalues and their eigenvectors are orthogonal.

Taking into account the important role of matrices in the theory of resonances, as well as the supposed importance of resonances for the genetic system, the scientific direction "matrix genetics" has been proposed in Russia (Petoukhov, 2008) and is developing with the participation of scientists from different countries. Now let us describe some new model approaches to use mathematical formalisms of the theory of resonances in the field of morphogenesis phenomena.

3. Curved biological surfaces and the concept of resonant genetics

Morphogenetic processes on different lines and branches of biological evolution sometimes demonstrate a surprising generality and various types of symmetry, including non-Euclidean symmetries. Examples of this generality are the law of homological series of N.I. Vavilov and the phenomena of phyllotaxis. This Section is devoted to using the notion of resonances of vibrational systems for modeling inherited curved surfaces on the basis of formalisms of differential Riemannian geometry and tensor analysis.

A characteristic feature of inherited biological surfaces is their curvilinear configurations, for example, curved surfaces of fruits, shellfish shells, animal and plant bodies. From the standpoint of mathematical modeling, this can be considered as inheritance of geometric characteristics of curved surfaces. The life of organisms is largely related to two-dimensional surfaces, for example, cell membranes and embryonic sheets that give rise to different organs and tissues (https://ru.wikipedia.org/wiki/Zerodyshevye_listki). In mathematics, curved surfaces are studied by means of differential Riemannian geometry and tensor analysis with using the key notion of the metric tensor (see, for example, Dodson and Poston (1991); Gallot et al., 2004; Rashevsky, 1964). Such metric tensor defines a metric in an infinitesimal part of the surface by specifying the distance between two of its infinitely close elements. The specification of the system (or the "field") of metric tensors on the surface determines its "internal" geometry, allowing it to calculate the arc lengths, the angles between the curves, and the areas of the regions on the surface, regardless of its spatial location. Therefore it is natural to try to create a general theory of biological morphogenesis with the use of metric tensors.

By definition, the metric tensor in an n-dimensional affine space with the introduced operation of scalar multiplication is given by a nondegenerate symmetric matrix $|| g_{ij} ||$, $g_{ij} = g_{ji}$ (Rashevsky, 1964, p. 157). In Riemannian geometry, 2-dimensional curved surfaces are described by means of metric tensors in their form of (2*2)-matrices. The symmetric real matrices of vibration systems with two degress of freedom satisfy to the definition of metric tensors and can be considered as metric tensors (Petoukhov, 2015a, 2016). There is an isomorphism between the set of (2*2)-matrices of vibrosystems and the set of metric tensors of 2-dimensional curved surfaces: for each matrix of the first set with its two orthogonal eigenvectors and two eigenvalues, there exists the matrix of the second set with the same eigenvectors and eigenvalues. Due to this isomorphism, mathematical models of Riemannian geometry for 2-dimensional curved surfaces of biological objects can be interpreted on the basis of language of theory of resonances of vibrational systems; the formal mathematical models obtain physical interpretations for further researches. It is one of main objectives of our study. More precisely, this isomorphism allows "encoding" or define metric tensors of curved surfaces through similar matrices of vibrosistems with two degrees of freedom, that is by means of the resonant frequencies of relevant vibration systems (the way of encoding morphogeneses through the resonant frequencies of the vibration systems). We use this isomorphism to model biomorphologic surfaces from the standpoint of the mentioned concept of resonance genetics. The coordinates g_{ij} of a metric tensor are the pairwise scalar products of reference vectors, on which it is constructed. If a square root is extracted from a symmetric matrix that is a metric tensor, then a new symmetric matrix is obtained whose columns represent these reference vectors (and which, in turn, can be treated as a new metric tensor). From such standpoint, hierachial systems of metric tensors exist, which can be considered as hierarchial systems of matrices of vibrational systems. In order for a symmetric real matrix to be interpreted as a tensor, it is necessary to specify the transformation group with respect to which it acts as a tensor. For example, for symmetric (2*2)-matrices considered in this paper, such a group is the group of rotations of the plane (or a wider group of motion transformations), the transformations of which leave the scalar products of the reference vectors invariable, although the coordinates of the vectors themselves are changed under these transformations.

Let us describe our model approach, which allows modeling curved biological surfaces and their growth transformations in a unified manner within the framework of the concept of resonant genetics. This approach proceeds from the statement about the key role of the resonant frequencies of vibration systems with two (and more) degrees of freedom for the genetic inheritance of morphological surfaces and their natural biological transformations. In this connection, this approach (or theory) is conditionally called the morphoresonance approach. It uses the isomorphism between the set of (2*2)- matrices of vibration systems and the set of metric tensors of two-dimensional curved surfaces embedded in a three-dimensional Euclidean space.

It is known that the metric tensor or metric at the point of the surface is represented in the form of the so-called first quadratic form from the differentials of the coordinates du, dv on the surface:

$$I = Edu^2 + 2Fdudv + Gdv^2, \quad (1)$$

where E, F, G are the coefficients of the form (the limited volume of our article doesn't allow reproducing here well known details of this part of Riemannian geometry, which are described, for example, in Dodson and Poston (1991); Gallot et al. (2004); Rashevsky (1964)). According to geometric meaning, this form coincides with the square of the element of arc length of the curve on the surface. In the orthogonal coordinate system (u, v) on the surface, the coefficient F = 0, and the expression of the form is simplified:

$$I = Edu^2 + Gdv^2, \tag{2}$$

These coefficients E and G are the eigenvalues of the metric tensor at a given point of the surface. They coincide with the eigenvalues of some (2*2)-matrix of a vibration system with two degrees of freedom, in view of the above isomorphism. Accordingly, these coefficients can be encoded (set) by the resonant frequencies of this vibration system.

The first quadratic form of the surface defines its internal geometry, but to characterize its curvature in space, the second quadratic form is used:

$$II = Ldu^2 + 2Mdudv + Ndv^2, \tag{3}$$

where L, M, N are the coefficients of this form. Its geometric meaning lies in the fact that it characterizes the deviation of the surface from the tangent plane at the point under consideration.

On curved surfaces there are many geometric types of lines: geodesic, asymptotic, etc. All of them can be analyzed to reveal their possible connection with the ideas of morphogenetic resonances and resonant genetics. But at the first stage of development of the morphoresonance approach, the central attention is paid to the main lines of curvature of surfaces. The network of these main lines of curvature forms an orthogonal system, which is convenient to use as a coordinate system. In construction mechanics, the theory of thin shells is created precisely on local coordinate systems on the basis of these main lines of curvature (Pogorelov, 2007, p. 162). We recall the information on curvatures and main lines of curvature.

The ratio II/I of the second quadratic form to the first form is called the normal curvature of the surface at a given point. It reaches its maximum and minimum values of k_1 and k_2 in two orthogonal directions, called the principal (we do not consider the special points, at which this value does not depend on the direction, for example, at points in the plane or sphere). The line of curvature on the surface is one that touches the principal direction at each of its points. If two orthogonal families of lines of curvature are used as the system of coordinate lines of the surface, the average coefficients of the first and second quadratic forms are zero: F = M = 0. The quantities of the principal curvatures k_1 and k_2 in the point are calculated via the ratio of the first and second quadratic forms from (2) and (3):

$$k_1 = L/E,\ k_2 = N/G \tag{4}$$

The specification of two quadratic forms defines a curved smooth surface. Of these, only the first form is positive definite and comparable to the positively determined (2*2)-matrices of vibration systems for specifing the internal geometry (ie, the metric) of the surface in this model approach. The coefficients of the second quadratic form, which is not positive definite in the general case, cannot be directly specified (encoded) through the resonance frequencies of such vibration systems. How, in our model approach, are the curvatures k_1 and k_2 connected with resonance frequencies? The answer to this question is given by expressions (4), in which these curvatures vary inversely under changes of the E and G coefficients of the first quadratic form. In this case, one can selectively change any of the curvatures by changing the resonance frequency, which determines the value of the corresponding coefficient of the first form. Indeed, under changes of the value of the resonance frequency corresponding to the coefficient E (or G) in the indicated matrix isomorphism, the curvature k_1 (or k_2) in the considered point of the surface selectively changes in line with (4) (assuming that the behavior of L and N in the numerators of these expressions is not significant). This resonance frequency of a vibration system with two degrees of freedom acts as a physical regulator of curvature at the point of the surface (given that the surface metric in a small neighborhood of the point is determined at once by both resonant frequencies of the given vibration system).

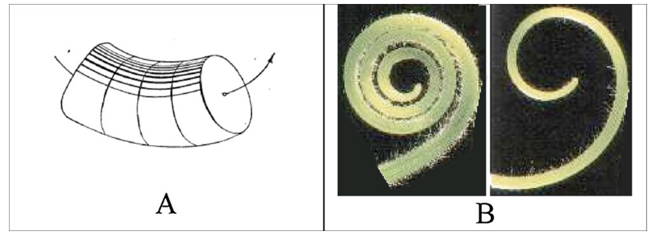

Fig. 1. Examples of channel surfaces in geometry (A) and in morphology of the growth transformation of a tendril of a plant (B).

An effective theory of morphological surfaces should be able not only to simulate static surfaces, but also the change of these surfaces in the course of growth and other natural transformations, when - at every moment of transformations - a new form is created that smoothly arises from the previous in a regular way. Our proposed theoretical approach is based on the supposition about the key role of resonant frequencies of vibration systems with two (and more) degrees of freedom for the genetic inheritance of morphologic surfaces and their natural biological transformations (Petoukhov, 2015a, b, 2016). This morphoresonance approach using the isomorphism between the set of (2*2)-matrices of vibration systems and the set of metric tensors of two-dimensional curved surfaces embedded in a three-dimensional Euclidean space. This approach seems to be appropriate for modeling many morphogenetic phenomena.

For an additional explanation of our approach, Fig. 1 shows examples of so called channel surfaces, which can be used for modeling many biological objects: blood vessels, plant shoots, etc. In geometry, channel surfaces are the surfaces formed by the motion of a circle of variable radius, at which the center of the circle moves along a given curve, and the plane of the circle remains perpendicular to this curve all the time (see mathematical definitions and equations of different channel surfaces in (Peternell and Pottmann (1997); https://en.wikipedia.org/wiki/Channel_surface). For channel surfaces, the generating circles are the main curvature lines, and the lines orthogonal to them are the main lines of curvature of the second family. For example, in the described model approach, the growth transformation of the tendril of the plant (Fig. 1) is interpreted as defined by a corresponding change in at least one of the two resonant frequencies that control the corresponding curvature k_1 or k_2 at the points of the surface. Of course, under changes of this regulating resonance frequency, not only the curvature but also the metric tensor of the channel surface is changed, which agrees with the phenomenology of the growth transformations under consideration. Thus, from the point of view of the morphoresonance theory, the transformation of some channel surfaces into others is primarily a smooth rearrangement of the network of principal curvature lines through the encoding assignment of one or two resonance frequencies of some regulating vibration system with two degrees of freedom.

One should add that networks of curvature lines of surfaces is conjugate to Möbius transformations known in geometry and physics. The latter transform the surfaces in such a way that the lines of curvature of the original surfaces go over into the lines of curvature of new surfaces (for comparison, projective-geometric transformations of surfaces transform the lines of curvature of initial surfaces into lines of a different kind). Mobius transformations (also called in the literature as conformal-geometric or circular transformations) preserve the angles, transfer spheres into spheres, and they are local-similar. Many phenomena of biological symmetries are built on these Mobius transformations and their cyclic groups in multiblock bodies (Petoukhov, 1989). Additional possible reasons for this biological realization of Mobius symmetries are their local-similar nature: the transformations of a small neighborhood of any point of the surface are scale, that is, preserving curvature lines with a simple scaling of the curvatures k_1 and k_2 at each point (the shape of the surface as a whole can vary significantly due to different scaling in its various points); from the point of view of the morphoresonance theory, this local similarity of transformations simplifies the resonance regulation of growth.

Another reason can be related to the important role of the network of main lines of curvature in the theory of thin shells of structural mechanics, where their use as local coordinate systems greatly simplifies the equations of mechanics connecting the shape of shells with stresses and deformations under loads (Pogorelov, 2007, p. 162). In the case of thin (momentless) hulls for a broad range of surfaces (axial symmetrical surfaces with an axial symmetrical load) the surface curvature lines coincidence with the lines of their main tensions: in other words, curvature lines are identified by the extreme mechanical properties. In biology the latter fact justifies the positioning along these lines of the centers of chemical interaction between the organic surface and the environment, because the extreme mechanical tensions probably have an extreme impact on the opening of micropores; on deformations of structural elements of chemical groupings on the surface; on renaturation and denaturation of collagen molecules, morphogenetically important; on biological rhythms in morphogenetic processes etc. The importance of mechanical stresses for morphogenetic processes was repeatedly emphasized in the writings of Beloussov and his associates (Beloussov, 2012, 2015; Cherdantsev, 2003; Cherdantsev, Grigorieva, 2012).

Concerning the theme of conformal (or Mobius) symmetries in biomorphology, we also recall that Maxwell's equations of electrodynamics are invariant under the group of Mobius transformations; the comprehension of this fact is one of the problems of physics, for the solution of which different authors proposed various versions of the conformal theory of relativity. The mathematical apparatus of quantum mechanics is also closely related to Mobius symmetries and there are several variants of conformal quantum field theory (for details, see (Petoukhov, 1981, Chapter 2)). Examples of modeling ontogenetic transformations of biological bodies based on Mobius transformations are shown in Fig. 2, which was reproduced by Beloussov in his book (Beloussov, 2015, p. 14) from the book (Petoukhov, 1988). These nonlinear morphogenetic transformations resemble the ideas of D'Arcy Thompson about the curvature of the space of biological bodies: he used curvilinear coordinates to demonstrate that the regular mathematical transformations of curvature of the body shape of an organism of one species reveals the relationship of its form to the body shape of an organism of a different kind (Thompson, 1942).

The tensor product of two metric tensors generates a new metric tensor related to the Riemannian space, respectively, of increased dimension. The same applies to the curvature tensors. By defining algorithms or rules for changing metric tensors and curvature tensors, one can build models of morphogenesis based on formalisms of Riemannian geometry and think about the following: the morphological similarity of generations are provided by the genetic transfer of the system of these tensors from generation to generation. In our model approach we believe that in bioinformatics and the biological evolution of organisms, the following hierarchial principle using the tensor product is realized to a certain extent: "systems of resonance frequencies encode more general systems of resonant frequencies", which includes the principle "metric tensors encode metric tensors of higher orders".

Physics and mathematics have gained valuable experience in the use of metric tensors, in particular, in connection with the notion of a tensor field. "*We say that we are given a tensor field if at each point M of space we are given a tensor of constant valence, but otherwise, generally speaking, it varying from point to point. This tensor will be called the field tensor*" (Rashevsky, 1964, p. 46). In physics, precedents are known when physical fields are identified with certain tensor fields. For example, Einstein and Grossman identified the gravitational field with the field of metric tensors of Riemann geometry. Since the Riemannian geometry is determined by the assignment of a doubly covariant symmetric tensor, any physical problem reducing to the study of such a tensor field can be formulated as a problem of Riemannian geometry. In particular, tensor fields of this type include various physical quantities characterizing the elastic, optical, thermodynamic, dielectric, piezomagnetic, and other properties of anisotropic bodies. In connection with the concept of resonant genetics, some of these physical applications of Riemannian geometry can be used to study the phenomena of biological morphogenesis and the development of the theory of the morphogenetic field.

In biology, for the explanation of the phenomena of morphogenesis long ago - from the very beginning of the 20th century - there are ideas of a certain morphogenetic field within the organism (Beloussov, 1997). Its versions are also known under other names: the embryonic field, the biological field, the cellular fields, etc.

The concept of resonant bioinformatics, taking into account the initial results obtained and the mathematical formalisms used (Petoukhov, 2015a, b, 2016; Petoukhov and Petukhova, 2017), leads to the emergence of a new version of the morphogenetic field, understood as the tensor field of frequency-resonant peculiarities and interactions in living matter of vibrational processes of different nature. The point is that in the living organism there are many types of interrelated vibrational and wave processes inscribed in the general picture of the genetic inheritance of its features: electromagnetic, electromechanical (because many biotissues are piezoelectrics), biochemical (accompanied by sometimes cyclic conformational changes in biological molecules), mechanical and etc. This set, increasing in the course of ontogeny of the organism, is also endowed with a growing system of resonant-frequency objects and their relationships, which largely determine the energy of various parts of the body and transfer of energy between them. When new portions of energy enter the body, for example, due to food from the outside world, this resonant system participates in their redistribution between organs and tissues, including ensuring morphogenetic processes. The proposed version of the morphogenetic field can be called a "morphoresonant field" (Petoukhov, 2015a,b, 2016).

The following primary definition is possible (Petoukhov, 2015a,b): a morphoresonant field is the tensor field of oscillatory or wave processes that exists within the body and develops in time with coordinated

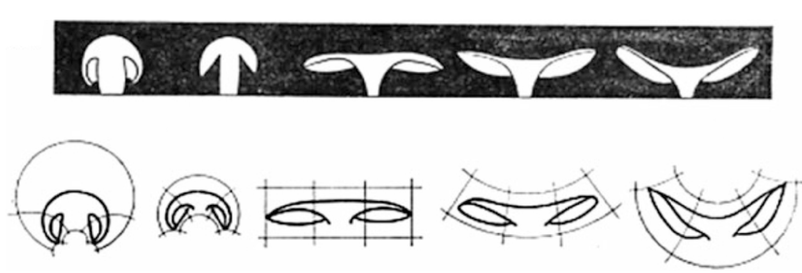

Fig. 2. Conformal symmetry transformations, preserving rectangular shapes of small parts under substantial deformations under growth of a mushroom fruit body (from Petoukhov, 1988; Beloussov, 2015).

resonant frequencies of the set of vibration systems with many degrees of freedom. In this version of the morphogenetic field, the existence of unknown types of physical fields is not assumed. Note that this version differs from the versions of other authors in that it - for the first time - connects the morphogenetic field with the mathematical formalisms of the known theory of resonances of oscillatory systems and also with the features of the molecular genetic system described in (Petoukhov, 2016). The versions of other authors - in their modeling approaches - did not connect morphogenetic phenomena with structures of molecular-genetic systems and with the mathematics of the theory of resonances. The theory of the morphoresonant field is aimed at the development of mathematical models of ontogenesis and phylogenesis of curvilinear biological forms on the basis of the provision on resonance mechanisms in living matter. From the standpoint of this model approach, morphogenesis is defined as the restructuring and development of the system of tensors of the morphoresonant field, or as the restructuring and development of a system of coordinated resonant frequencies of oscillatory processes in the body. The non-linear deformation of the biological body is connected with the action of the so-called vibrational and wave forces described in vibrational and wave mechanics, where many amazing phenomena exist: phenomena of vibrational separation and structuring of multiphase media, vibro-transportation of substances, vibro-transfer of energy and so forth (Blekhman, 2000; Ganiev et al., 2015). Practically invisible vibrations can provide, for example, the following phenomena: the upper position of the inverted pendulum becomes stable; heavy metal ball “floats” in a layer of sand; a rope takes a form of a vertical stem if a corresponding vibration acts on its base. Inside fluids, vibrating bodies can attract or repel each other (vibrating forces of Bjerknes) and pulsating gas bubbles may coalesce or divide. Some applications of these phenomena to model processes in biological objects are discussed in (Petoukhov, 2016).

In embryology, it is known that serious embryonic abnormalities (for example, microsurgical removal of its parts) in many cases does not prevent it from growing into a completely normal organism (Beloussov, 1998, 2015). What is the criterion for the correctness of the finite configuration of the organism growing out of the embryo? Such a criterion can be based on the coordination of the system of resonant characteristics of an adult organism with the resonant patterns of its molecular genetic system. The ability of resonance processes to influence structural formation is known in physics by the example of the figures of Chladni and cymatics (Jenny, 2007). In them, for demonstration of the resonance shaping, external vibration is applied to a resonant plate coated with powdery particles. These examples don't exhaust at all the form-forming potentials of resonance systems, especially in cases where vibration is not imposed from the outside, but each particle of the ensemble itself vibrates actively with its own oscillation parameters and is in resonance interactions with other particles.

Mutual synchronization of morphogenetic processes is important for a living body. Vibrational mechanics gives the known example of resonant self-synchronization of plurality of oscillating pendulums mounted on a common movable platform (Harvard demonstration – http://www.youtube.com/watch?v=Aaxw4zbULMs). Inside a living organism, its structural water apparently plays the role of such common mobile platform, which is required for synchronization. Illustrative example of morphogenetic and general physiological role of structural water is given by jellyfish, which consists of 99% water, but despite of this its morphology implements heritable phyllotaxis phenomena: tentacles, canals and zooids of some jellyfish exactly correspond to phyllotaxis laws (Jean, 1994, Chapter 12.3.3). This structural water is also a candidate for the role of a unifying vibro-platform for vibro-transfer of energy among different parts of a living body. The physical features of structural water, associated with resonance interactions in it, are currently being studied in laboratories around the world. An important role in vibro-connections among parts of an organism belongs also to cytoskeleton that works in coordination with boundary water and membranes (Igamberdiev, 2012).

4. Phyllotaxis and the mathematical theory of resonances

Let us give an example how matrix mathematics of the theory of resonances can be connected with model approaches to morphogenetic phenomena of phyllotaxis. Usually, the laws of phyllotaxis are described as such inherited spiral arrangements of leafs in plants that are characterized by the numbers of the Fibonacci series ($F_{n+2}=F_n+F_{n+1}$, $F_0 = 0$, $F_1 = 1$). But similar laws dictate also inherited configurations not only of plant organisms, but also alpha-helices of polypeptide chains, parts of the body of animals, including shells of mollusks, etc. (see reviews in the books (Jenny, 2007; Petoukhov, 1981)). In other words, the laws of phyllotaxis appear in inherited morphological structures at very different levels and branches of biological evolution.

One of known model approaches, which was not connected with theory of resonances till now, uses so called Fibonacci matrix Q that bear the characteristic name - "the matrix of growth" - in theories of phyllotaxis (Jenny, 2007). The exponentiation of this matrix in integer powers yields matrices, all entries of which are Fibonacci numbers:

$$Q=\begin{vmatrix}0 & 1\\ 1 & 1\end{vmatrix};\ \ Q^n = \begin{vmatrix}F_{n-1} & F_n\\ F_n & F_{n+1}\end{vmatrix} \tag{5}$$

In connection with the mentioned concept of resonance genetics, let us pay attention to the eigenvalues of this symmetric "matrix of growth" (5), which have not been considered in mathematical and theoretical biology before. Table 1 shows that the eigenvalues of the Fibonacci matrix Q are equal to the famous golden section $\varphi = 0{,}5^*(1 + 5^{0,5}) = 1618\ldots$ and its reciprocal value φ^{-1} with a minus sign. This connection of the golden section with the eigenvalues of the matrix Q can be used for the new definition of the golden section via this matrix of growth. The golden section is a mathematical symbol of a self-reproduction for many centuries (Leonardo da Vinci, *J.*Kepler, etc). It is known that many authors note the golden section in different physiological systems: cardio-vascular system, respiratory system, electric activities of brain, *etc*. For the theory of resonances of vibration systems, in which matrices should have positive eigenvalues (since they are equal to the square of the resonant frequency), the square of the Fibonacci matrix Q^2 is appropriate and interesting, because in the matrix Q^2 both eigenvalues φ^{-2} and φ^2 are positive (Table 1).

Both eigenvalues φ^{-2n} and φ^{2n} of any of matrices Q^{2n} (Table 1) are mutually inverse and determine the point with the Cartesian coordinates (φ^2, φ^{-2}) on the hyperbola $y = 1/x$ (one can note that this hyperbola plays the key role in the concept of resonance genetics for modeling the principal psychophysiological law of Weber-Fechner (Petoukhov, 2016)). From the point of view of the concept of resonances, these data on the eigenvalues of even degrees of the Fibonacci matrix Q^{2n} indicate the possibility of modeling the morphogenetic laws of phyllotaxis in the language of resonances of vibration systems with two degrees of freedom in the close analogy with the mentioned resonance model of the Weber-Fechner psychophysical law. One can think that living matter skillfully uses hyperbolic structures in different situations, giving them some universal character (recall that hyperbolic rotations and hyperbolic numbers are expressed by symmetric matrices that can be treated as matrix representations of the corresponding

Table 1
The eigenvalues and eigenvectors of the Fibonacci matrix Q and of its even powers Q^{2n} (n = 1, 2, 3, …).

Matrix	Their eigenvalues	Eigenvectors
Q = [0 1; 1 1]	$-\varphi^{-1}$, φ	[-φ, φ°] and [φ°, φ] [a]
Q^2 = [1 1; 1 2]	φ^{-2}, φ^2	[-φ, φ°] and [φ°, φ] [a]
$(Q^2)^n$ = [F_{2n-1} F_{2n}; F_{2n} F_{2n+1}]	φ^{-2n}, φ^{2n}	[-φ, φ°] and [φ°, φ] [a]

[a] Note: the eigenvectors in this table are given without their normalization per unit length. When normalized to a unit length, each of them has a normalizing coefficient $k = (\varphi^2 + 1)^{-0.5} = 0.5257\ldots$.

vibration systems).

In vibration systems, the eigenvalue φ^2 corresponds to the resonance frequency φ. But is anything known about the existence in nature of vibration systems, whose resonant frequencies are associated with the golden section φ? Yes, it is. The journal "Science Daily" has published an article with the characteristic title "The Golden Section is Open in the Quantum World" (Coldea et al., 2010) about the following. Researches of cobalt niobate, which has magnetic properties, have revealed: "*the chain of atoms acts like a nanoscale guitar string. … The tension comes from the interaction between spins causing them to magnetically resonate. For these interactions we found a series of resonant notes: the first two notes show a perfect relationship with each other. Their frequencies (pitch) are in the ratio of 1.618…, which is the golden ratio famous from art and architecture*".

Modeling of phyllotaxis laws in the language of resonant frequencies can be continued further (Petoukhov, 2015a). For example normalized eigenvectors of the Fibonacci matrix Q can be considered as columns of the matrix M = [-φ*k, k; k, φ*k], which is the unitary matrix and which is also the reflection matrix since $M^2 = E$ (here $k = (\varphi^2 + 1)^{-0.5} = 0.5257\ldots$; E is the identity matrix). This fact is interesing because matrices of such types play an important role in quatum mechanics and quantum computung. But let us turn now to the question, which was especially interesting for Beloussov, about the role of photons in inherited morphogenetic phenomena (Beloussov et al., 2007; Voeikov and Beloussov, 2007).

5. Photons, photonic crystals and genetic information for morphogenesis

From the point of view of quantum mechanics, the interaction of molecules is based on the emission and absorption of photons with the participation of resonance correspondences. Therefore, special attention should be paid to the important role of photons and photonic crystals in genetic informatics.

As known, a photon is a type of elementary particle, the quantum of electromagnetic field including electromagnetic radiation such as light, and the force carrier for electromagnetic field (in particle physics, force carriers or messenger particles or intermediate particles are particles that give rise to forces between other particles (https://en.wikipedia.org/wiki/Force_carrier). A photon has two possible polarization states. Photon energy is the energy carried by a single photon. The amount of energy is directly proportional to the photon's electromagnetic frequency. Photon energy is solely a function of the photon's frequency.

Photons, which are radiated by different molecular elements, can differ by their frequencies. In DNA and RNA, each of their nitrogenous bases – adenine A, cytosine C, guanine G, thymine T and uracil U – posesses individual traits from the following sets of binary-oppositional traits or indicators: purine or pyrimidine, strong or weak hydrogen bonds, amino or keto (see more details in (Petoukhov, 2008, 2016; Petoukhov, He, 2010)). From the standpoint of quantum mechanics, each of these molecular indicators can emite photons with its own individual frequencies for interactions with other molecules on the bases of the emission and absorption of photons. Since these photons have their individual frequencies, they can be named as color photons. Correspondingly for modeling aims, one can consider the appropriate set of color photons emitted by the mentioned molecular indicators. In our model approach, these color photons provide actions of the molecular indicators of the nitrogenous bases onto surrounding molecules to transfer genetic information. From these point of view the existence of DNA sequences is accompanied by a rich set of appropriate beams of special color photons with different energy to provide a cooperative information functioning of ensembles of genetic elements. Taking this into account, a quantum-algorythmic model approach was proposed for molecular genetics (Petoukhov, 2017; Petoukhov, Svirin, 2018).

From this point of view, nitrogenous bases A, C, G, T/U and their combinations in DNA and RNA are resonance determinants of frequencies of genetic photonic ensembles within living bodies (or briefly, "geno-photon determinants"). The reading and transmission of genetic information from DNA and RNA molecules occurs by means of a set of resonance frequencies of their photons. DNA encodes quantum states of its photon beams. The photons language is a serious candidacy for the role of a basic language of molecular-genetic information. Figuratively speaking, from this point of view, life in its information aspects is woven from the light.

Photons are actively studied in modern science as elements of quantum computers and devices of quantum cryptography. In models of quantum computers, conventional light polarizers are used to create pure and mixed states of n-qubit systems of light beams. The idea of ensembles of «multicolor» photons for a creation of n-qubit states, which was noted by us above in the connection with DNA-texts, was independently used in the recent engineering work of Canadian scientists (Caspani et al., 2016). This work has revealed a new perspective way to create quantum computers. For increasing dimensionality of the photon quantum state, its authors used the ability to generate multiple photon pairs on a frequency comb, correpsponding to resonances in specifically designed microcavities. Such technological achievemnets can be useful for deeper understanding the role of beams of multicolor photons in genetic informatics and in inherited morphogenetic phenomena.

Modern engineering technologies actively use so-called photonic crystals to control the spatial distribution of photon beams (Joannopoulos et al., 2008; https://en.wikipedia.org/wiki/Photonic_crystal). A photonic crystal is a periodic optical nanostructure that affects the motion of photons. Photonic crystals contain regularly repeating regions of high and low dielectric constant. Photons (behaving as waves) either propagate through this structure or not, depending on their wavelength. This gives rise to distinct optical phenomena, such as inhibition of spontaneous emission, high-reflecting omni-directional mirrors, and low-loss-waveguiding. The periodicity of the photonic crystal structure must be around half the wavelength of the electromagnetic waves to be diffracted. One should note that, as known, living bodies posses inherited opportunities to manage photonic beams using physical principles of photonic crystals with their properties of photon gratings, etc. Many inherited biological phenomena of structural coloration and of animal reflectors are built on this, including a beautiful coloring of butterfly wings, peacock feathers, etc. (see details and lists of references in https://en.wikipedia.org/wiki/Photonic_crystal, https://en.wikipedia.org/wiki/Animal_reflectors, https://en.wikipedia.org/wiki/Structural_coloration). It is natural to assume that the genetic transfer of inherited properties of photonic crystals in biological bodies is built on that the molecular genetic structures themselves possess the properties of photonic crystals. One can remind here the Schrödinger's definition of chromosomes as aperiodic crystals (Schrödinger, 1944).

We believe that spatial characteristics of ensembles of genetic and other biological molecules, that form complex diffraction structures, play the managing role of photonic crystals in the problem of controlling photon beams that are generated and absorbed by these molecules (the range of photon frequencies in living bodies can be very wide, far beyond the optical range). In particular, the spatial configuration of genetic molecules as photonic crystals is an important factor in controlling the processes of transmission of genetic information from DNA and RNA molecules with using photon beams generated by them.

In our opinion, the inherited morphogenetic processes in living bodies are also determined to a large extent by biological photon beams, the course of which is not accidental, but is strictly organized by a system of spatial characteristics of ensembles of genetic and other biological molecules as photonic crystals. In the course of ontogeny, on the basis of electromagnetic (photonic) interactions, new molecular materials are involved into a naturally growing biological body, which leads to the appropriate growth of the managing system of biophotonic crystals and to the growth of numbers of photon beams. Of course, quantum-mechanic laws of resonances in molecular photonic interactions play the key role. On this basis, we develop our concept of the "morpho-resonance field" to model morphogenetic phenomena (Petoukhov, 2015a, b, 2016).

The phenomenon of vibro-transfer of energy among parts of an oscillatory system is known: a rotary electromotor operates stably, when it is disconnected from the power electrosupply, if it is standing on a mutual vibro-platform with another rotary electromotor of similar resonant characteristics, which is connected to a power supply (self-synchronization by resonant interactions). Taking into account possibilities of such energy transferring, living organisms can be seen as resonance consumers of energy of surrounding electromagnetic waves coming from space and the depths of the earth. Photosynthesis, which playes a huge role in providing the biological life and which is based on absorbing solar energy of light waves, is probably only one of examples of the biological consumption of energy from external wave sources on the basis of resonant mechanisms (a resonant "vampirism" of energy and information in organisms).

Classical electrodynamics describes a photon as an electromagnetic wave with its circular right or left polarization. To the theme “life and photons”, one can add many interesting connections of these polarization properties of photons with inherited properties of living bodies, for example, the following:

- One of the biggest mysteries of nature is the asymmetry of biological molecules, accompanied by a preferred direction of the rotation - to the left or to the right - of the polarization plane of light by these molecules (this was discovered by Louis Pasteur). For example, all biological amino acids (except the simplest glycine, which is symmetric), from which the proteins of all living organisms are composed, exist only in one of two possible asymmetric forms - in the left form. Amino acids in this form rotate the plane of polarization of light to the left. Our body doesn't use amino acids with the opposite right form, rotating the plane of polarization of light to the right. Biological catalysts - enzymes -, being built asymmetrically, act only on one optical antipode, without touching the another. The same asymmetry with respect to the right and left is inherent not only in amino acids, but also in the nucleotides that form DNA and RNA. The reason for this is the asymmetry of the components of the sugar, which is part of the nucleotides and which provides the optical activity of DNA and RNA molecules: they rotate the plane of polarization of light to the right.
- Millions of species of living organisms (insects, mollusks, arthropods, etc.) are endowed with inherited ability to see in polarized light (a human organism does not possess this ability).

Deep investigations of the role of photons and photonic crystals in genetic informatics and morphogenesis can lead to many new discoveries and useful applications.

6. Some concluding remarks

Deep works by L. Beloussov in the field of fundamental problems of morphogenesis gave rise to a wide range of works by other authors. Our own works can be considered as one of their continuations. This review article shows our data about the important role of resonances and photonic crystals in genetic informatics. Mathematical formalisms of differential Riemannian geometry and tensor analysis are used for modeling inherited curved surfaces in biomorphology. Notions of a morpho-resonance field are discussed. The connection of the golden section with the Fibonacci matrix of growth used in morphogenetic models of phyllotaxis is shown.

A living organism is a single whole. A creation of integral approaches to biological phenomena is one of main task of modern mathematical and theoretical biology (see, for example, (Integral biomathics, 2012; Simeonov, 2013; Simeonov et al., 2017). The concept of resonant genetics (or resonant bioinformatics) allows modeling - in a single academic language of matrix mathematics - biological phenomena of different levels and different areas of physiology: from molecular genetics to the morphogenetic phenomena and inherited psychophysical laws. From this standpoint, the body can be considered as a complex oscillating part of the universe, associated with the vibrational processes of the outside world by resonant relationships. The account of these resonant relationships is also useful in the problems of weak and superweak influences in biology and medicine.

One can mention that – in line with thoughts of some authors – morphogenesis defines geometric structures not only of our body but also of our mind. Here the following thoughts can be quotated from (Nalimov, 2015, p. 115): “*artificial intelligence could be brought closer to mathematical thinking if it were possible to realize the metrical properties of the human mind. … the consciousness itself is structured geometrically: any person in his existential aspects is geometric. … in our minds, when constructing texts through which we perceive the World, something very similar to what happens in morphogenesis occurs. We are ready to see in the depths of consciousness the same geometric images that are revealed in morphogenesis*”.

Genetic molecules belong to the microworld and therefore are subordinated to the principles of quantum mechanics. Quantum mechanics operates with frequency and resonance characteristics of quantum-mechanical objects; its mathematics uses eigenvalues of matrices. In general, quantum mechanics was emerged and developed largely as a science about resonances in microworld. Thus, the concept of system-resonance genetics creates models of genetic phenomena on the same language of frequencies and resonances, on which models in quantum mechanics are based. In addition to this, it uses the same matrix language, on which "matrix mechanics" of Werner Heisenberg has been created; it is historically the first form of quantum mechanics, which retains its value to this day.

Many authors supposed that living organisms use principles of quantum computers. For example, in his thoughts about quantum computers in living organisms, R. Penrose appeals to the known fact that tubulin proteins exist in two different configurations, and they can switch between these configurations like triggers to provide bio-computer functions (Penrose, 1996). By contrast to this “protein standpoint”, results of the model approach of the resonance genetics testify that already the molecular-genetic level, which is the deepest level of living organisms, is connected with the principles of quantum computers (Petoukhov, 2017, 2018; Petoukhov and Svirin, 2018).

The concept of resonance genetics can facilitate a convergence of biology and quantum mechanics, possibility of which is studied by many authors (see for example (Igamberdiev, 2014; Matsuno, 1999, 2003; Matsuno, Paton, 2000; Patel, 2001a, b)). It proposes a new class of mathematical models for biological symmetries in genetics and inherited morphogenetic structures. The creator of the theory of resonances in structural chemistry L. Pauling was right when he supposed an important meaning of resonances in organization of living matter (Pauling, 1940).

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