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RESONANCES, WALSH FUNCTIONS AND LOGICAL HOLOGRAPHY IN GENETICS AND MUSICOLOGY

Petoukhov S.V.[1], Petukhova E.S.[2]

1 - Biophysics, bioinformatics (b. Moscow, Russia, 1946).

Address: Laboratory of Biomechanical Systems, Mechanical Engineering Research Institute of Russian Academy of Sciences; Malyi Kharitonievskiy pereulok, 4, Moscow, 101990, Russia. E-mail: spetoukhov@gmail.com.

Fields of interest: genetics, bioinformatics, biosymmetries, multidimensional numbers, musical harmony, mathematical crystallography (also history of sciences, oriental medicine).

Awards: Gold medal of the Exhibition of Economic Achievements of the USSR, 1974; State Prize of the USSR, 1986; Honorary diplomas of a few international conferences and organizations, 2005-2012.

Publications and/or Exhibitions: 1) S.V. Petoukhov (1981) *Biomechanics, Bionics and Symmetry*. Moscow, Nauka, 239 pp. (in Russian); 2) S.V. Petoukhov (1999) *Biosolitons*. *Fundamentals of Soliton Biology*. Moscow, GPKT, 288 pp. (in Russian); 3) S.V. Petoukhov (2008) *Matrix Genetics, Algebras of the Genetic Code, Noise-immunity*. Moscow, RCD, 316 pp. (in Russian); 4) S.V. Petoukhov, M. He (2010) *Symmetrical Analysis Techniques for Genetic Systems and Bioinformatics: Advanced Patterns and Applications*, Hershey, USA: IGI Global, 271 pp.; 5) He M., Petoukhov S.V. (2011) Mathematics of Bioinformatics: Theory, Practice, and Applications. USA: John Wiley & Sons, Inc., 295 pp.

2 – Computer scientist (b. Moscow, Russia, 1987).

Address: Laboratory of Biomechanical Systems, Mechanical Engineering Research Institute of Russian Academy of Sciences; Malyi Kharitonievskiy pereulok, 4, Moscow, 101990, Russia. E-mail: spetoukhov@gmail.com.

Fields of interest: computer sciences, bioinformatics, biosymmetries, psychology.

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Music is a game with acoustic resonances, to which people are remarkably predisposed. Throughout tens of thousands of years, people create musical instruments, adjusting them to specific systems of resonances. Over the centuries, people have learned to combine individual instruments and singers into orchestras and choirs as coordinated oscillating systems with an increased number of degrees of freedom. An organism can be seen as a musical synthesizer with multiple settings of inherited resonant modes [Petoukhov, 2015a,b, 2016a]. Gottfried Leibniz declared that music is arithmetic of soul, which computes without being aware of it. Taking into account that music is represented by systems of resonances, one can reformulate this declaration: systems of resonances are the arithmetic of soul, which computes without being aware of it. The notion "resonance" is one of the most fundamental one in classical and quantum mechanics. E. Schrodinger emphasised the basic meaning of resonances: "*The one thing which one has to accept and which is the inalienable consequence of the wave-equation as it is used in every problem, under the most various forms, is this: that the interaction between two microscopic physical systems is controlled by a peculiar law of resonance*» [Schrodinger, 1952, p.115]. In our opinion all materials described below are closely related with resonant mechanisms in bio-molecular systems.

Analysis of symmetric organization of DNA-alphabets and some genetic phenomena has showed that the genetic system is related to matrix mathematics of resonances of oscillatory systems with many degrees of freedom [Petoukhov, 2016a,b, 2016a]. Our results testify in favor that molecular-genetic systems are organized on ensembles of resonances and that binary sets of resonances are the base of binary genetic computers. It gives materials for a new proposed slogan: a living body is a musical instrument (a synthesizer with an abundance of rearrangements of resonant modes). These results are related with the hierarchy of Fibonacci-stage scales of "genetic music" described in [Darvas et al., 2012; Petoukhov, 2008a, 2015a; Petoukhov, He, 2010]. These scales were revealed due to study of binary oppositions in molecular-genetic system and they have two types of intervals by analogy with the Pythagorean musical scale (unlike equal temperament).

The paper is devoted to further mathematical study of binary-oppositional structures of molecular-genetic system. Due to these results one of the authors of the article (S.Petoukhov) has declared and has argumented that DNA- and RNA-structures show the existence in living bodies a system of geno-logical coding (or briefly "geno-logical code"), which encodes biological systems of Boolean functions with using Walsh functions and Walsh-Hadamard spectra. The geno-logical code exists in parallel with the genetic code of amino acids. To model this system of geno-logical coding, mathematical approaches from engineering theory of digital devices are applied including dyadic groups of binary numbers and spectral logic of systems of Boolean functions. Correspondingly a new class of bio-mathematical models and research methods is proposed, in particularly, models of the genetic logical holography on the base of Walsh functions. On this way, algebra-logical biology is under development. Let us briefly describe initial bases of this scientific field.

Binary numbers and logical operations with them are the foundation of computers. To use binary numbers, Boolean functions and logical operations, computers contain two-positional switches (triggers), conditions of which determine numbers 0 or 1. In physiology, the law “all-or-none” for excitable tissues is known long ago: for example, a nerve cell or muscle fiber give only their answers "yes" or "no" under action of different stimulus. If a stimulus is below a certain threshold, a nerve or muscle fiber give no response. If a stimulus is above a certain threshold, a nerve or muscle fiber give its response of the maximal amplitude [Kaczmarek, Levitan, 1987; Martini, 2005]. In other words, it works in the alternative mode - "yes" or "no" - by analogy with Boolean variables and logic functions. A separate muscle, which contains many muscle fibers, can reduce its length in different degree due to the combined work of the plurality of its muscle fibers. Nervous system can also account stimulus of a different force due to the combined action of its many nerve fibers (and also due to the ability to change the frequency of the generation of nerve impulses at their fixed amplitude). In his thoughts about quantum computer foundations of physiological phenomena, R. Penrose [1996] appeals to known fact that tubulin dimers (tubulin proteins are a base of microtubules in living bodies) exist in two different configurations and they can switch between these configurations like triggers.

All these known facts testify that a living organism can be interpreted as a genetically inherited huge network of triggers of different types, different biological level, different functionality and material incarnation, including trigger subnets of tubulin proteins, muscle fibers, neurons, etc. From this perspective, biological evolution can be represented as a process of self-organization and self-development of systems of biological trigger networks. Correspondingly, the Darwinian principle of natural selection is understood, first of all, as natural selection of biological networks of triggers together with appropriate systems of Boolean functions, on which a coordinated work of these networks being built. In light of this it is naturally to think that the genetic system, which provides transmission of corresponding logic networks along the chain of generations, can be also organized on principles of binary numbers and their algebra of logic. In this case the trigger network of genetic molecules participates in the genetic realization of all other trigger networks of the body, and - in this sense – this network is the most fundamental, which defines algorithms and relationships of all the trigger networks of a whole organism.

Dyadic groups of binary numbers and Walsh functions in genetics

Genetic molecules are closely related with binary numbers. The basic alphabet of DNA consists of four polyatomic letters – adenine A, cytosine C, guanine G, thymine T (uracil U in RNA) of a very simple structure. The set of these four structures is not quite heterogeneous, but it carries on itself the symmetric system of binary-oppositional traits. The system of such traits divides the genetic four-letter alphabet into various three pairs of letters, which are equivalent from a viewpoint of one of these molecular traits or its absence (Fig. 1): 1) C=T & A=G (according to the binary-oppositional traits: "pyrimidine" or "purine"; 2) A=C & G=T (according to the traits: amino or keto); 3) C=G & A=T (according to the traits: three or two hydrogen bonds are materialized in these complementary pairs, that is strong or weak bonds) (Gumbel et al., 2015; Petoukhov, 2008a; Stambuk, 1999). Below we use traditional denotations of these traits: purine – R, pyrimidine – Y, amino – M, keto – K, strong hydrogen bonds – S, weak hydrogen bonds – W.

TRAITS	**G**	**A**	**C**	**T(U)**
1) purine (R), pyrimidine (Y)	R	R	Y	Y
2) amino (M), keto (K)	K	M	M	K
3) strong hydrogen bonds (S), weak hydrogen bonds (W)	S	W	S	W

Fig. 1. Three binary sub-alphabets according to three kinds of binary-oppositional traits in a set of nitrogenous bases C, G, A, T(U). Left: the molecular structure of these bases in DNA. Right: the partition of the four-letter alphabet of DNA (RNA) into three binary sub-alphabets in accordance with three binary-oppositional traits marked by symbols R, Y, M, K, S and W.

As a result we obtain the representation of the 4-letter alphabet as the system of three "binary sub-alphabets corresponding to these attributes"; in each of binary sub-alphabets, each of 4 letters can be denoted by 0 or 1. Each of traits of nitrogenous bases A, C, G, T(U) in Fig. 1 can be interpreted as connected with its own resonance characteristics. For example, it is obvious that purines may have resonance characteristics that differ from the resonance characteristics of pyrimidines due to differences in the structure of the purine and pyrimidine molecules. In this light, each of mentioned pairs of binary-oppositional traits can be treated as a pair of oppositional resonance characteristics or, in other words, as the pair of oppositional resonant traits, which can be used for genetic binary computers on the base of resonances [Petoukhov, 2016a].

For binary numbers in computers and in the theory of discrete signal processing, the fundamental logic operation is modulo-2 addition. By definition, the modulo-2 addition of two numbers written in binary notation is made in a bitwise manner in accordance with the following rules: $0 \oplus 0 = 0$, $0 \oplus 1 = 0 \oplus 1 = 1$, $1 \oplus 1 = 0$ ($\oplus$ is the symbol for modulo-2 addition). For example, modulo-2 addition of two binary numbers 110 and 101, which are equal to 6 and 5 respectively in decimal notation, gives the result $110 \oplus 101 = 011$, which is equal to 3 in decimal notation. By means of modulo-2 addition, binary n-bit numbers form so called dyadic groups, including 2^n members. For example, the set of binary 3-bit numbers

$$000, 001, 010, 011, 100, 101, 110, 111 \qquad (1)$$

forms a dyadic group, in which modulo-2 addition serves as the group operation [Harmuth, 1989]. The distance in this symmetry group is known as the Hamming distance. The modulo-2 addition of any two binary numbers from a dyadic group always results in a new number from the same group. The series (1) is transformed by modulo-2 addition with the binary number 001 into a new series of the same numbers: 001, 000, 011, 010, 101, 100, 111, 110. Such changes in the initial binary sequence, produced by modulo-2 addition, are termed dyadic shifts [Ahmed and Rao, 1975; Harmuth, 1989]. If any system of elements demonstrates its connection with dyadic shifts, it indicates that the structural organization of its system is related to the logic of modulo-2 addition. The works [Petoukhov, 2008a, 2016b] show a few interesting results of applications of dyadic shifts and matrices of dyadic shifts to study structures of genetic alphabets.

The second fact of a connection of genetics with binary numbers and their dyadic groups is that DNA- and RNA-alphabets of n-plets contain the same quantities of members as in dyadic groups of 2n-bit binary numbers (n = 1, 2, 3,…), for example:

- the DNA-alphabet of 1-plets contains 4 members (A, C, G, T) by analogy with 4 members of the dyadic group of 2-bit binary numbers (00, 01, 10, 11);

- the DNA-alphabet of doublets contains 16 members (AA, AC, AG, …, TT) by analogy with 16 members of the group of 4-bit binary numbers (0000, 0001, …, 1111);

- the DNA-alphabets of triplets contains 64 members (AAA, AAC, …, TTT) by analogy with 64 members of the dyadic group of 6-bit binary numbers (000000, 000001, …, 111111).

From mathematical standpoint, dyadic groups of binary numbers are closely related with systems of Walsh functions, which are characters of dyadic groups; groups of Walsh functions are isomorphic to appropriate dyadic groups [Fine, 1949; Harmuth, 1977; Karpovsky, Stankovic, Astola, 2008]. But Walsh functions have been revealed in structures of molecular-genetic phenomena [Petoukhov, 2005b, 2008a,b]. (Rademacher functions, which were noted in these publications, are a particular case of Walsh functions). These data give the third evidence of a connection of genetics with dyadic groups. One can remind that Walsh functions coincide with rows of Hadamard matrices $[1\ 1;\ 1\ -1]^{(n)}$, where (n) is Kronecker power. Hadamard matrices play important roles in many tasks of discrete signal processing; tens of thousands of publications devoted to them (see a review in [Seberry, 2005]). For example, codes based on Hadamard matrices have been used on spacecrafts «Mariner» and «Voyager», which allowed obtaining high-quality photos of Mars, Jupiter, Saturn, Uranus and Neptune in spite of the distortion and weakening of the incoming signals. Hadamard matrices are used in quantum computers ("Hadamard gates") and in quantum mechanics in the form of unitary operators, etc. [Ahmed, Rao, 1975; Seberry, Wysocki, Wysocki, 2005]. Walsh functions play basic role in the sequency analysis [Hartmut, 1977, 1981], which is one of important types of spectral analysis in communication technologies on discrete signals. For our theme of the geno-logical code it is important that the spectral logic of discrete devices is based on Walsh functions and Walsh-Hadamard transformation [Karpovsky, Moscalev, 1973; Karpovsky, Stankovic, Astola, 2008; Zalmanzon, 1989].

The importance of the connection of genetics with dyadic groups is enhanced by the fact that binary numbers are the basis of Boolean algebra logic on which computers operate. Thoughts about analogies between functioning of living organisms and technical computers exist long ago (see for example [Elsevier, 2014; Hameroff, Penrose, 1996, 2014; Igamberdiev, Shklovskiy-Kordi, 2016; Ji, 2012; Liberman,1972; Penrose, 1996]). We add here our results about analogies between molecular genetics and computers and also about the connection of genetics with Walsh functions and Hadamard matrices, which lead to our idea about the geno-logical code on the base of spectral logic.

As known, for a creation of a computer, usage of material substances for its hardware is not sufficient but logical operations should be also included, which can successfully work in different hardware from very different materials. The same situation is true for living bodies, where genetic systems should provide genetic information not only about material substances (proteins) but also about logic of interrelated operations in biological processes. One can think that the genetic code of amino acids defines proteins in biological bodies and the geno-logical code defines logic rules and functions of their operating work. Both biological codes exist in parallel and complement each other.

Dyadic analysis in musicology

The new kind of mathematics in modeling genetic phenomena gives possibilities of new heuristic associations and new understanding of the natural phenomena. Mathematics of dyadic groups, dyadic spaces and spectral logic has much specificity including notions of lattice functions, logical operations with them, dyadic convolution, dyadic derivatives, etc. It gives new mathematical tools not only for genetics but also for musicology. It may be recalled that musical structures are associated with dyadic groups through the traditional dichotomy of their time intervals. Fig. 1 illustrates that in the case of the dichotomous division, 2^n equal parts of a whole time interval correspond to a sequence of 2^n members of dyadic groups of binary numbers. It is one of arguments to apply mathematics of dyadic groups and spectral logic in musicology.

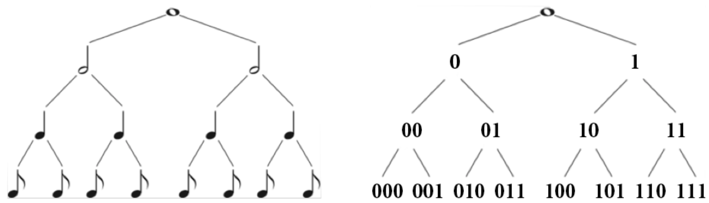


Figure 1. Dichotomous trees of notes values and dyadic groups of binary numbers

Comparative analysis in musicology is widely used to study musical works of different authors, different styles, different eras, and so on. Mathematical tools of a dyadic analysis and spectral logic, which we introduce in the field of genetics, may be also useful in these tasks of musicology, since the functioning of living organisms is coordinated with genetic structures and algorithms. One can hope to achieve on this way a better understanding of musical creativity and its relationship with the principles of the genetic code, which is still non-studied. Let us show an example of application in musicology the notion of the dyadic derivative, which is defined by the following expression [Gibbs, 1967; Stankovic, Astola, 2008]:

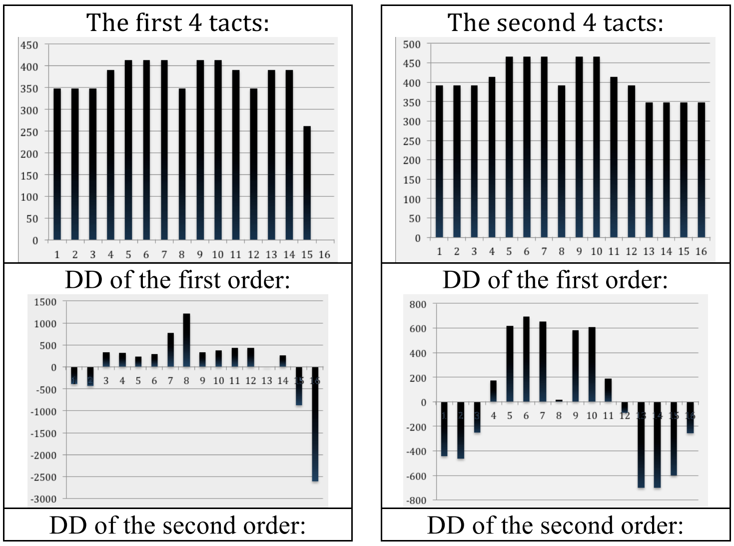
$$f^{[1]}(x) = -\frac{1}{2}\sum_{r=0}^{n-1}(f(x \oplus 2^r) - f(x))2^r \qquad (1)$$

where $f^{[1]}(x)$ is the dyadic derivative of the first order, $f(x)$ is an analyzed lattice function (2^n-dimensional vector). By contrast to ordinary derivative of a curve, which characterizes the behavior of a curve in one point, the dyadic derivative characterizes the behavior of a whole 2^n-dimensional vector f(x). Dyadic derivatives relate to symmetrology since dyadic derivatives of symmetrical vectors (with internal translational and mirror symmetries) are also vectors of the same type of symmetry. They are closely related with Walsh functions [Stankovic, Astola, 2008].

Fig. 2 shows the set of the first 8 tacts of the known Russian song "Katyusha", which is used here only as an example of application of dyadic derivatives for a comparison dyadic analysis of 2^n-dimensional vectors in musicology.

Figure 2. The fragment of the known Russian song "Katyusha".

The shortest duration of notes in this fragment of the song is equal to 1/8, therefore each tact with its duration 2/4 can be represented as a 4-dimensional vector, if each note is represented, for example, by a numeric value of its frequency in Hertz. In this case the whole fragment of the song is represented as the 32-dimensional vector [348,348,348,391,414,414,414,348,414,414,391,348,391,391,261,0,391,391,391,414,465,465,465,391,465,465,414,391,348,348,348,348]. One can make a comparison dyadic analysis of 2^n-dimensional parts of this vector. For example, Fig. 3 shows such representation of the first 4 tacts of the song and of its second 4 tacts as 16-dimensional vectors; dyadic derivatives of two first orders for both vectors are also shown for their comparison.



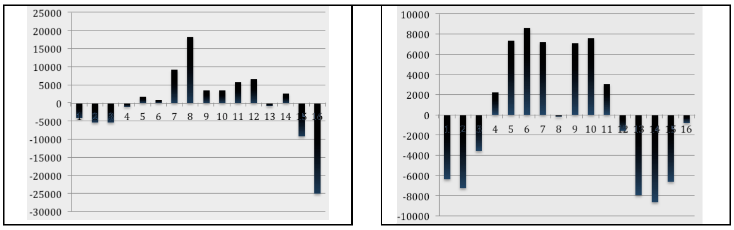


Figure 3. Diagrams of vector representations of the first 4 tacts (on left) and the second 4 tacts of the song "Katyusha" (Fig. 2) with frequencies in Hertz. Their dyadic derivatives (DD) are also shown.

By analogy different mathematical tools of dyadic analysis can be also applied in other biological fields, where natural sequences allow their representations in a form of 2^n-dimensional vectors: genetic sequences, graphics in medical diagnostics (for example, cardio-logical curves), poetry and other literary texts, design and so forth.

Genetics and the logical holography by Walsh functions

Living organisms possess properties, which seem to be analogical to properties of holography with its nonlocality of information. For example, German embryologist Hans Driesch separated from each other two or four blastomeres of sea urchin eggs or on changing their mutual positions. The main result of Driesch's experiments was that fairly normal (although proportionally diminished) larvae with all of their organs properly arranged could be obtained from a single embryonic cell (blastomere) containing no more than 1⁄2 (if two first blastomeres were separated) or even 1⁄4 (in the case of four blastomeres separation) of the entire egg's material. Rather soon these effects (defined by Driesch as "embryonic regulations") were numerously confirmed and extended to the species belonging to almost all taxonomic groups of metazoans, from sponges to mammalians [Belousov, 2015; Driesch, 1921]. These experimental results testify that complete sets of "causes" required for further development are contained not only within whole eggs/embryos but also in their halves, quarters, etc. The similar properties exist in holograms, where one can restore a whole holographic image of a material object from a part of the hologram. A hologram has such property since each element of the hologram possesses information about all elements of the represented object (in difference to ordinary photo).

Such nonlocality information can be modelled in the field of digital signal processing under the following conditions concerning a transformation of multi-dimensional vectors: 1) all entries of an initial vector are represented in each entry of a transformed vector; 2) the transformed vector can be restored unambiguously into the initial vector. Both of these conditions are satisfied in the case of 2^n-dimensional vectors, which are transformed by appropriate Hadamard matrix H (rows of which are Walsh functions). For example, if a 4-dimensional vector $\bar{X}$ = [x_0, x_1, x_2, x_3] is transformed by the symmetrical Hadamard matrix H_4 = [1,1,1,1; 1,-1,1,-1; 1,1,-1,-1; 1,-1,-1,1], each entry of the transformed vector $\bar{X}$*H_4 contains all entries of the vector $\bar{X}$:

$[x_0, x_1, x_2, x_3]*H_4 = [x_0+x_1+x_2+x_3, x_0-x_1+x_2-x_3, x_0+x_1-x_2-x_3, x_0-x_1-x_2+x_3]$. For restoring the initial vector $\bar{X}$ from the transformed vector $\bar{X}*H_4$, the following multiplication is made: $(\bar{X}*H_4)*H_4/4 = \bar{X}$ (here the basic property of Hadamard matrices H_n is used: $H_n*H_n^T = n*E$, where H_n^T is the transformed Hadamard matrix, E is identity matrix, n – the order of the matrix H_n).

Nonlocality of information is demonstrated not only in embriology but also in brain functions including associative memory, physiological processing visual information, etc. Analogies between brain functions and holography are described in many publications (see for example [Greguss, 1968; Pribram, 1971]). For example, due to associative memory, a short fragment of a musical piece can evoke in our mind the whole musical piece. But the brain and the nervous system have appeared at a relatively late stage of biological evolution. A great number of species of organisms is living without neuronal networks. It is clear that the origins of the similarity between holography and nonlocal informatics of living organisms need to search at the level of the genetic system.

Physical holography, which possesses the highest properties of noise-immunity, is based on a record of standing waves from two coherent physical waves of the object beam and of the reference beam. But physical waves can be modeled digitally. Correspondingly noise-immunity and other properties of optical and acoustical holography can be modeled by digitally, in particular, with using Walsh functions and logic operations concerning dyadic groups of binary numbers. This can be made on the base of discrete electrical or other signals without any application of physical waves. The pioneer work about «holography by Walsh waves» was [Morita, Sakurai, 1973]. The work was devoted to Walsh waves (or Walsh functions), which propagate through electronic circuits - composed of logical and analog elements - by the analogy with the optical Fourier transform holography. In this digital Walsh-holography, objects, whose digital holograms should be made, are represented in forms of 2^n-dimensional vectors. Due to application of Walsh-Hadamard transformation, information about such vector is written in each component of the appropriate hologram, which is also a 2^n-dimensional vector, to provide nonlocal character of storing information.

This digital Walsh-holography under the title «logical holography» was also considered later in [Derzhypolskyy, Melenevskyy, Gnatovskyy, 2007; Golubov, Efimov, Skvortsov, 1987; Soroko, 1974]. All these and other works about logical Walsh-holography considered possibilities of its apllications only in engineering technologies of digital signals processing without any supposition of its application in biology, in particular, in genetics. Our previous publications [Petoukhov, 2008a,b, 2016a,b] described deep connections of the genetic code systems with Walsh functions, Hadamard matrices, dyadic groups and logical modulo-2 addition. On the base of these results and some other materials, the hypothesis has been put forward that genetic system is related with principles of logical holography and appropriate logic operations [Petoukhov, 2016b, Appendix C]. This hypothesis leads to a new class of mathematical models of genetic structures and phenomena on the base of logical holography and logical operations. Correspondingly we develop the theory of «genetic logical holography», where mathematics of logical holography is used for modeling genetic and other physiological phenomena.

The algorithm of the logical holography operates with object vectors and reference vectors by analogy with classical holography, which is based on interference of object waves with reference waves. This algorithm consists of two parts, both of which use Walsh functions and Walsh-Hadamard transformations: 1) generation of a logical hologram of a vector-signal; 2) restoration of the vector-signal from the logical hologram [Soroko, 1974].

On this algorithm, the model of a zipper-like reproduction of DNA molecules has been proposed [Petoukhov, 2016b, Appendix C]. As known, replication of genetic material in DNA serves to preserve genetic information in a series of successive generations of cells and organisms. Replication of DNA strands takes place before each division of normal DNA-containing structures in eukaryotes (nuclei, mitochondria and plastids), and also before each division of bacterial cells and before each reproduction of DNA-viruses. Then doubled DNA in the process of segregation is distributed equally between the two daughter cell nuclei or bacterial cells. The process of replication of genetic information and its subsequent dichotomous segregation equally between child objects can be repeated many times as you like. In this scheme the nature provides a very high degree of noise-immunity of the transmitted information, which can be compared with the highest noise-immunity of information in holographic recordings. As known, the replication of DNA resembles a zipper mechanism with its teeth, which may cling to each other or unzipp from each other step by step. In our mentioned model (Fig. 4), each of nitrogenous bases C, G, A, T is represented in a form of a 4-dimensional vector related with Boolean functions: C = [1, 0, 0, 0]/2, G = [0, 0, 1, 0]/2, A = [0, 1, 0, 0]/2, T= [0, 0, 0, 1]/2. More details of the model one can see in the original work.

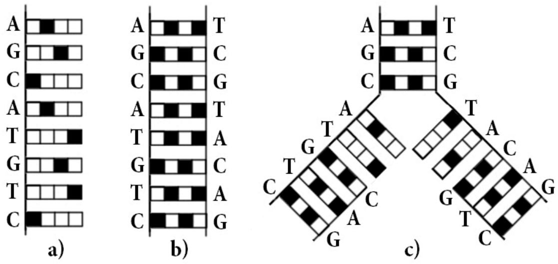


Figure 4. The illustration to the model of the zipper-like reproduction of DNA [Petoukhov, 2016b, Appendix C]. Nitrogenous bases of DNA are denoted as informational vectors: C=[1,0,0,0]/2, G = [0,0,1,0]/2, A = [0,1,0,0]/2, T = [0,0,0,1]/2. These vectors are shown schematically as structures with 4 cells in each, where each black cell means 0.5 and each white cell means 0. a) A separate DNA-chain; b) two complementary chains of DNA; c) unzipping two chains of DNA.

Another proposed model of genetic phenomena simulates the known fact that, in nucleotide sequences of DNA and RNA, a great number of different repetitions exists including complementary palindromes and simple repetitions [Gusfield, 1997; Lehninger, 1982]. For instance, families of repetitive sequences occupy about one-third of the human genome. The importance of the problem of repeats in genetic sequences is reflected in the fact that 6000 articles was published on this subject during 20 years before 1991 [Gribskov, Devereux, 1991]. Musicology also known many examples of different repetitions of fragments in musical pieces including simple repetitions (or reproductions) of tacts of an identical duration along musical pieces (Fig. 2 shows one of examples).

One of many applications of classical holography is devoted to the reproduction of numerous copies of flat templates required for the production of printed circuits, etc. [Wenyon, 1978]. In this case a special holographic operation is used, which has its analogue in mathematics under the title "convolution of two functions". For example, if such convolution of a flat image of a triangle with images of three points is made, the result is an image of three triangles (Fig. 5).

Figure 5. Illustration of reproduction of numerous copies of flat templates (a triangle in this case) by means of technology of classical holography (after [Wenyon, 1978]).

In logical holography the analogue of this operation of reproduction of copies is the dyadic convolution Z(m) of two 2^n-dimensional vectors $\overline{X}$ and $\overline{Y}$, which is known in the theory of discrete signal processing and which is calculated by means of the following expression, where ⊕ means logical modulo-2 addition [Ahmed, Rao, 1975]:

$$Z(m)=n-1 * \sum_{h=0}^{n-1} X(h)Y(m\oplus h),\ m=0,1,\ldots,n-1 \qquad (2)$$

Let us consider one example to explain replications in vectors by means of the dyadic convolution (2) of two 2^n-dimensional vectors in logical holography. In the spare vector $\overline{X}$ = [A, G, C, T, 0, 0, 0, 0], where A, G, C, T are symbols of real numbers, we call non-zero part A,G,C,T as the "template" (or "kernel"), which can be reproducted in various variants by means of its dyadic convolution with different variants of the reproducing vector $\overline{Y}$ (Fig. 6). In general case this template may be very different in its length and in

$\overline{Y}$	Result of the dyadic convolution $\overline{X}$ and $\overline{Y}$
[8, 0, 0, 0, 8, 0, 0, 0]	[A, G, C, T, A, G, C, T]
[8, 0, 0, 0, 0, 8, 0, 0]	[A, G, C, T, G, A, T, C]
[8, 0, 0, 0, 0, 0, 8, 0]	[A, G, C, T, C, T, A, G]
[8, 0, 0, 0, 0, 0, 0, 8]	[A, G, C, T, T, C, G, A]

Figure 6. Replications of the template A,G,C,T of the vector $\overline{X}$ = [A, G, C, T, 0, 0, 0, 0] in the result of its dyadic convolution with different reproducing vectors $\overline{Y}$

its composition. The reproducing vector $\overline{\Upsilon}$ is an analogue of the image of points in the mentioned example from classical holography about reproduction of image of a single triangle into its many copies (Fig.5). One can see from Fig.6 that different orders of elements of the template A,G,C,T in its copies are possible in dependence of the reproducing vector $\overline{\Upsilon}$.

In the case of long vectors, this method of logical holography allows modeling long sequences, in which an initial template is repited many times in different parts of the sequences (see details in [Petoukhov, 2016b, Appendix C]). In songs, in other musical pieces and in poetry we also meet repetitions, which can be simulated and stydied by means of this mathematical approach, which can be used also for a creation of algorythmical music.

Fibonacci numbers, Boolean functions and the geno-logical coding

Theorists of music and aesthetic proportions have repeatedly drawn attention to the connection of many musical creations of known composers, and also features of musical perception, with the golden section and Fibonacci numbers. One can see the review of this material, for example, on the website http://www.goldenmuseum.com/. Fibonacci-stage scales (tunings) lie in the base of the new direction of musical culture - genetic music, which is intensively developed in the Moscow P.I. Tchaikovsky Conservatory [Koblyakov, Petoukhov, Stepanyan, 2016].

Fibonacci numbers F_n (Fig. 7) satisfy the recurrence relation $F_{n+1}=F_n+F_{n-1}$, and the ratio F_{n+1}/F_n tends to the golden section $f = (1+5^{0.5})/2 = 1{,}618\ldots$ with increasing n.

F_n	0	1	1	2	3	5	8	13	21	34	55	89	144	…
n	0	1	2	3	4	5	6	7	8	9	10	11	12	…

Figure 7. The series of Fibonacci numbers.

In biology, Fibonacci numbers are known long ago as numeric characteristics of phyllotaxis laws of morphogenesis, which are shown on different levels and branches of biological evolution [Jean, 1994; Kappraff, 2004].

In mathematics, so called Fibonacci matrix Q = [1 1; 1 0] exists, which has the following property related with Fibonacci numbers: $Q^n = [F_{n+1}, F_n; F_n, F_{n-1}]$. The Fibonacci matrix Q sometimes is called as the "growth matrix" since it was used many times in biomathematics for simulation of phyllotaxis phenomena [Jean, 1994].

In the beginning of our paper we said about the geno-logical code, mathematics of which is based on the spectral logic of systems of Boolean functions and which determines many inherited processes in living bodies including morphogenesis. From this standpoint it is natural to think that Fibonacci numbers and the Fibonacci matrix Q are related with the geno-logical code and correspondingly they have interesting mathematical interrelations with formalisms of such spectral logic described in [Karpovsky, Stankovic, Astola, 2008; Zalmanzon, 1989]. We reveal that it is true and such interrelations really exist. Let us show some of them. Below we use the following terms: vectors, all entries of which are Fibonacci numbers, are called "Fibonacci vectors"; vectors, all entries of which are Boolean variables 0 and 1, are called "Boolean vectors".

Rows of the Fibonacci matrix Q are Boolean vectors [1 1] and [1 0]. Multiplication of the [2*2]-matrix Q^n with any of Boolean 2-dimensional non-zero vectors [0, 1], [1, 0] or [1, 1] gives a Fibonacci vector (Fig. 8).

$[0, 1]*Q = [1, 0]$	$[0, 1]*Q^2 = [1, 1]$	$[0, 1]*Q^3 = [2, 1]$	$[0, 1]*Q^4 = [3, 2]$	…
$[1, 0]*Q = [1, 1]$	$[1, 0]*Q^2 = [2, 1]$	$[1, 0]*Q^3 = [3, 2]$	$[1, 0]*Q^4 = [5, 3]$	…
$[1, 1]*Q = [2, 1]$	$[1, 1]*Q^2 = [3, 2]$	$[1, 1]*Q^3 = [5, 3]$	$[1, 1]*Q^4 = [8, 5]$	…

Figure 8. Multiplications of Boolean vectors with the Fibonacci matrix Q^n

One can see from Fig. 8 that an additive series of Fibonacci vectors $V_{k+1}=V_k+V_{k-1}$ arises (Fig. 9) by analogy with the series of Fibonacci numbers (Fig. 8). In each of vectors V_k sum of squares of its components is equal to a Fibonacci number: $F_n^2 + F_{n+1}^2 = F_{2n+1}$.

V_k	[1, 0]	[1, 1]	[2, 1]	[3, 2]	[5, 3]	[8, 5]	[13, 8]	[21, 13]	…
k	0	1	2	3	4	5	6	7	…

Figure 9. The additive series of 2-dimensional Fibonacci vectors V_k .

And what about Walsh-Hadamard spectra of members of this series of Fibonacci vectors V_k relative to Hadamard matrix H=[1 1; 1 -1] that is, in other words, what type of vectors arises in the result of multiplication V_k*[1, 1; 1, -1]? The answer is that Walsh-Hadamard spectra of Fibonacci vectors V_k are new Fibonacci vectors W_p, which form a new additive series $W_{p+1}=W_p+W_{p-1}$ (Fig. 10). In each of vectors W_p sum of squares of its components is equal to a Fibonacci number with factor 2.

W_p	$V_0*H=[1, 1]$	$V_1*H=[2, 0]$	$V_2*H=[3, 1]$	$V_3*H=[5, 1]$	$V_4*H=[8, 2]$	…
p	0	1	2	3	4	…

Figure 10. The additive series of Fibonacci vectors W_p, which are Walsh-Hadamard spectra of Fibonacci vectors V_k from Fig. 9.

Dyadic derivates (1) of Fibonacci vectors V_k and W_p give new Fibonacci vectors $D(V_k)$ and $D(W_p)$, which form new additive series of Fibonacci vectors (in the case of $D(V_k)$, Fibonacci vectors have factor 1/2) as Fig. 11 shows.

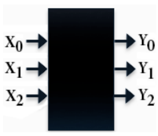
V_k	[1, 0]	[1, 1]	[2, 1]	[3, 2]	[5, 3]	[8, 5]	[13, 8]	..
$D(V_k)$	[1, -1]/2	[0, 0]/2	[1, -1]/2	[1, -1]/2	[2, -2]/2	[3, -3]/2	[5, -5]/2	..
W_p	[1, 1]	[2, 0]	[3, 1]	[5, 1]	[8, 2]	[13, 3]	[21, 5]	..
$D(W_p)$	[0, 0]	[1, -1]	[1, -1]	[2, -2]	[3, -3]	[5, -5]	[8, -8]	..

Figure 11. Dyadic derivates $D(V_k)$ and $D(W_p)$ of vectors V_k and W_p from Fig. 9, 10.

The wide theme of interrelations between 2^n-dimensional Fibonacci vectors and their dyadic derivates needs to be published separately. One should mention that the Fibonacci (2*2)-matrix Q can be generalized into Fibonacci matrices of higher orders, for example, by means of Kronecker multiplications of Q with the matrix [1, 1; 1, 1] [Petoukhov, 2003].

And how Fibonacci vectors are related with the spectral logic of systems of Boolean functions, which is described in [Karpovsky, Stankovic, Astola, 2008; Zalmanzon, 1989]? The spectral logic considers digital devices with input channels X_0, X_1,…X_n and output channels Y_0, Y_1, …, Y_m. Signals in each of these channels can be only 0 or 1 (Boolean variables). Fig. 12 (left) shows a simple example of such digital device and two examples of a typical tabular representation of systems of Boolean functions Y_0, Y_1 and Y_2. For each of 8 possible combinations of values of input Boolean variables X_0, X_1, X_2, each of output Boolean functions Y_0, Y_1 and Y_2 takes one of the possible values 0 or 1. Such correspondence is written in rows of the table. This system of Boolean functions is characterized by one 8-dimensional vector S, which is called a step-function and each component of which is equal to decimal notation of 3-bit binary number $Y_0Y_1Y_2$ in the same row. Walsh-Hadamard spectra of such step functions of systems of Boolean functions (switching functions) are studied in the spectral logic as key characteristics for analysis and synthesis of digital devices.

The first table (Fig. 12, middle) shows a case of digital devices with the system of only two non-zero Boolean functions Y_1 and Y_2, step function of which is the Fibonacci vector S_1=[3, 2, 0, 0, 0, 0, 0, 0]. Its Walsh-Hadamard spectrum S_1*$[1, 1; 1, -1]^{(3)}$ is equal to the Fibonacci vector [5, 1, 5, 1, 5, 1, 5, 1]. The second table (Fig. 12, right) shows a case of digital devices with the system of three non-zero Boolean functions Y_0, Y_1 and Y_2, the step function of which S_2 = [5, 3, 0, 0, 0, 0, 0, 0] is the Fibonacci vector with increased values of its non-zero components. Its Walsh-Hadamard spectrum S_2*$[1, 1; 1, -1]^{(3)}$ is equal to another Fibonacci vector [8, 2, 8, 2, 8, 2, 8, 2].



X_0	X_1	X_2	Y_0	Y_1	Y_2	S_1
0	0	0	0	1	1	**3**
0	0	1	0	1	0	**2**
0	1	0	0	0	0	**0**
0	1	1	0	0	0	**0**
1	0	0	0	0	0	**0**
1	0	1	0	0	0	**0**
1	1	0	0	0	0	**0**
1	1	1	0	0	0	**0**

X_0	X_1	X_2	Y_0	Y_1	Y_2	S_2
0	0	0	1	0	1	**5**
0	0	1	0	1	1	**3**
0	1	0	0	0	0	**0**
0	1	1	0	0	0	**0**
1	0	0	0	0	0	**0**
1	0	1	0	0	0	**0**
1	1	0	0	0	0	**0**
1	1	1	0	0	0	**0**

Figure 12. Examples of representations of digital devices in the spectral logic.

The Fibonacci step functions S_2 is obtained from the Fibonacci step function S_1 by means of the transformation $S_2 = S_1*Q_8$, where $Q_8 = ||q_{ij}||$ (here i,j = 1, 2,.., 8) is the Fibonacci (8*8)-matrix with the following entries: q_{11}=1, q_{12}=1, q_{21}=1, all other entries are equal to zero.

It is obvious that for a case of a Fibonacci vector with higher values, for example, of the vector with Fibonacci numbers 55 and 34, we need to appeal to the system with more than three Boolean functions Y_0, Y_1 and Y_2 since the decimal value of the maximal 3-bit binary number 111 is equal to 7. Decimal number 55 is equal to 6-bit binary number 110111. Correspondingly we need a system with 6 Boolean functions Y_0, Y_1, Y_2, Y_3, Y_4 and Y_5 to represent number 55 as a component of a step function in tables of the spectral logic.

From the standpoint of the spectral logic, one can model Fibonacci phyllotaxis phenomena on the base of Fibonacci step functions (or their spectra) of genetic systems of Boolean functions. Iterative actions of Fibonacci matrices on Fibonacci vectors lead to increasing non-zero values in new and new Fibonacci vectors. To represent these increasing values in step functions of the spectral logic, systems with more and more quantities of Boolean functions should be organized. Such algorithmic growth of systems of Boolean functions can be interpreted as a part of ontogenesis of phyllotaxis structures in living bodies. We believe that the manifestation of Fibonacci numbers on very different levels and branches of biological evolution is related with the geno-logical coding. Fibonacci matrices, which are sometimes called the matrices of growth, act as matrices of breeding of systems of Boolean functions. Fibonacci systems of Boolean functions define some archetypes of physiologic patterns, including phyllotaxis.

In this approach, morphogenetic and other onto- and phylogenetic phenomena are interpreted as consequences of an algorithmic growth of genetic systems of Boolean functions. In engineering technologies, the spectral logic of systems of Boolean functions has wonderful achievements of analysis and synthesis of digital devices, which have elements of artificial intellect for a detection and a correction of errors; an adaptation to the external environment; a training in a course of work; an interrelation with other digital devices, etc. All these possibilities can now be transferred into the field of study and mathematical modeling of genetically inherited biological phenomena including bio-rhythmic processes and the ability of newborn animals to coordinated movements on the base of inherited motion algorithms without training (the genetic biomechanics). This transfer became possible due to the identification of the structural kinship of molecular-genetic systems with Walsh functions and mathematical formalisms of the spectral logic of systems of Boolean functions. It is this kinship is the foundation of our doctrine about geno-logical coding in living matter, which allows developing spectral-logical genetics by analogy with the theory of digital devices.

Similar formalisms of the spectral logic can be applied in the field of musicology, including genetic music on the base of Fibonacci-stage scales.

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