Symmetry: Culture and Science
Vol. 26, No. 3, page_379-397, 2015

RESONANCES AND GENETIC BIOMECHANICS

S.V. Petoukhov

Biophysicist, bioinformatist (b. Moscow, Russia, 1946).

Address: Laboratory of Biomechanical Systems, Institute of Machine Science of Russian Academy of Sciences; Malyi Kharitonievskiy pereulok, 4, Moscow, 101990, Russia. E-mail: spetoukhov@gmail.com.

Fields of interest: genetics, bioinformatics, biosymmetries, multidimensional numbers, musical harmony, vibrational mechanics, mathematical crystallography (also history of sciences, oriental medicine).

Awards: Gold medal of the Exhibition of Economic Achievements of the USSR, 1974; State Prize of the USSR, 1986; his inclusion by the Chinese government in the official "List of Outstanding Scientists in the World" in 2012; Honorary diplomas of a few international conferences and organizations, 2005-2014.

Publications and/or Exhibitions: 1) S.V. Petoukhov (1981) *Biomechanics, Bionics and Symmetry*. Moscow, Nauka, 239 pp. (in Russian); 2) S.V. Petoukhov (1999) *Biosolitons. Fundamentals of Soliton Biology*. Moscow, GPKT, 288 pp. (in Russian); 3) S.V. Petoukhov (2008) *Matrix Genetics, Algebras of the Genetic Code, Noise-immunity*. Moscow, RCD, 316 pp. (in Russian); 4) S.V. Petoukhov, M. He (2010) *Symmetrical Analysis Techniques for Genetic Systems and Bioinformatics: Advanced Patterns and Applications*, Hershey, USA: IGI Global, 271 pp.; 5) He M., Petoukhov S.V. (2011) Mathematics of Bioinformatics: Theory, Practice, and Applications. USA: John Wiley & Sons, Inc., 295 pp.

Abstract: *Love to music is associated with a remarkable ability of living bodies to realizing their own resonance tunings. This ability is genetically inherited; vocal singing and speech communication are realized on it, in particular. Has the structural organization of the genetic coding any connection with mathematical formalisms of the theory of resonances in oscillation systems with many degrees of freedom? This article is devoted to the study of this issue. It shows possibilities of simulation of number of features of genetic phenomena and structures by means of these known formalisms. Our results testify in favor of a deep connection of genetics of living matter with special tensor families of resonances. The conception of resonant genomes is proposed.*

Keywords: symmetry, harmony, resonance, genetic code, genetic biomechanics, matrix analysis, vibrational mechanics.

1. INTRODUCTION

Music is a game with systems of acoustic resonances, to which human organisms are surprisingly predisposed. From the ancient time people creates musical instruments with different systems of resonances, musical usage of which causes human emotions. The human brain does not possess a special center of music. The feeling of love to music seems to be dispersed in the whole organism. The scientific studies of physiological mechanisms of musical perception took place long ago [Weinberger, 2004].

Living organisms are endowed with an innate ability to tune in to resonances, and they can use resonant frequencies as carriers of information, for example, in human vocal singing and speech communication. One might think that this inherited ability is based on a fundamental fact of existence of a living organism in the form of a huge choir of agreed oscillatory processes (mechanical, electromagnetic, etc..); this choir is capable to significant complication in the course of the ontogenetic development of the organism. Since ancient times chronomedicine claims that all diseases are caused by disorders of this harmony in the choir. From a formal point of view, a living organism is an oscillating system with a large number of degrees of freedom, and with the development of an organism from embryo to adult the number of its degrees of freedom is greatly increased while maintaining the coherence of oscillatory processes at each stage of development. Natural to assume that the mathematical modeling of such inherited choir of coordinated vibrational bioprocesses, which are connected with the genetic coding, is useful for in-depth understanding of the inherited ability of the body to music systems of resonances.

How to simulate and study this inherited coherence of biological oscillatory processes? To study of oscillatory systems with n-degrees of freedom, the theory of oscillations uses matrix apparatus of simulation of resonance features based on n-dimensional vector spaces. But whether exists in mathematics of matrices a type of operations that allows simulation of such inherited oscillatory processes with increasing dimension of vector spaces? Yes, such is the well-known operation of tensor matrix multiplication, which in this paper is used to model biological systems of inherited resonances associated with molecular-genetic structures and phenomena. All natural objects possess resonance properties. Whether exists in living organisms some distinctive specificity in their inherited systems of resonance characteristics, which is connected with genetic phenomena and structures of molecular-genetic coding? This article presents a model approach, the results of studies of which testify in favor of the specificity of biological systems of inherited sets of resonances. This model approach is based on a relatively narrow class of systems of resonances that is connected with eigenvalues and eigenvectors of (2^n*2^n)-matrices from tensor families based on the tensor (or Kronecker) product of (2*2)-matrices.

1. MATRIX REPRESENTATIONS OF RESONANCES AND THE TENSOR PRODUCT OF MATRICES

Matrices possess a wonderful property to express resonances, which sometimes is called as their main quality [Balonin, p. 21, 26]. Physical resonance phenomenon is familiar to everyone. The expression y=A*S models of transmition of a signal S via an acoustic system A. If an input signal “y” is a resonant tone, then the output signal will repeat it with a precision up to a scale factor y = λS by analogy with a situation when a string sounds in unison with the neighboring vibrating string. In the case of a matrix A, the number of resonant tones S_i corresponds to its size. They are called its eigenvectors, and the scale factors λ_i with them are called its eigenvalues or, briefly, a spectrum A**.** Frequencies $\omega_i = \lambda_i^{0.5}$ are called as natural frequencies of the system, and the corresponding eigenvectors are called as its own forms of oscillations (or simply, natural oscillations). These free undamped oscillations occur in the system in the absence of the friction forces in it and in the absence of external excitation forces. Behavior of the system in conditions of free oscillations determines by its behavior in many other conditions. In this context, one of the main tasks the theory of oscillations is a determination of natural frequencies (mathematically, the eigenvalues of the operators) and the natural forms of oscillations of bodies. To find all the eigenvalues λ_i (i.e. spectrum system A) and eigenvectors of the matrix A, which are defined by the matrix equation A*s = λ*s, the "characteristic equation" of the matrix A is analyzed: det(A-λE)=0, where E - the identity matrix. The characteristic equation together with its eigenvalues and eigenvectors is fundamental in the theory of mechanical, electrical and other oscillations at macroscopic or microscopic levels. The theory of oscillatory objects with many degrees of freedom uses systems of differential equations that can be represented in a concise and convenient matrix form of a symmetric type [Gladwell, 2004]. Symmetric matrices have real eigenvalues and orthonormal eigenvectors. Most matrices that are relevant to problems in the theory of oscillations are symmetric [Gladwell, 2004, p. 178].

This paper considers the spectra of (2^n*2^n) matrices, which are generated in the result of tensor products of initial (2*2) matrices and which are used for modeling genetic phenomena and structures. The tensor product of matrices, denoted by ⊗, is widely used in mathematics, physics, informatics, control theory, etc. It is used for algorithmic generation of higher dimensional spaces on the basis of spaces with smaller dimensions (reminding a growth of degrees of freedom in the ensemble of cells of growing organism in the result of their division). By definition, the tensor product of two square matrices V and W of the orders m and n respectively is the matrix $Q = V \otimes W = ||v_{ij} * W||$ with the order m*n [Bellman, 1960]. The tensor product has the property of inheritance of mosaic structure of the original matrix under its tensor exponentiation. This property connects the operation with fractals [Gazale, Chapter X]. Fig. 1 shows an example of the formation of fractal patterns, the type of which depends on the mosaic of the original matrix.

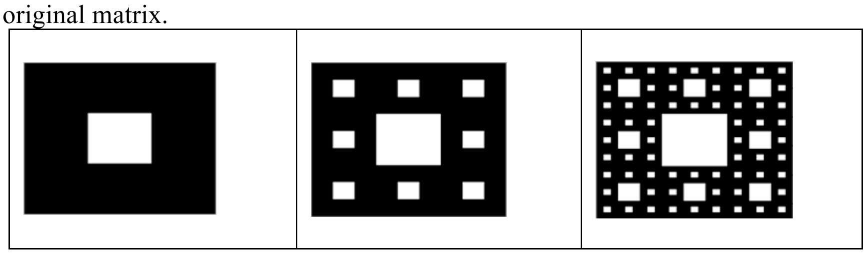


Figure 1: fractal patterns of Sierpinski carpet, which are formed due to tensor exponentiation of the matrix M = [1, 1, 1; 1, 0, 1; 1, 1, 1] in the second and third powers $M^{(2)}$ $M^{(3)}$ (shown in the center and right). Black and white colors in the matrices correspond to elements 1 and 0.

2. TABLE OF INHERITANCE OF EIGENVALUES OF MATRICES AND PUNNET SQUARES IN GENETICS

The tensor product of matrices is also endowed with the property of "inheritance" of their eigenvalues: if the original matrix V and W have the eigenvalues λ_i and μ_j respectively, then all the eigenvalues of the tensor product of $Q = V \otimes W$ are equal to $\lambda_i * \mu_j$ (figuratively speaking, λ_i and μ_j are inherited in this tensor way).

Features of tensor inheritance of eigenvalues of the original matrices (or "parental" matrices) matrices in the result of their tensor product can be conveniently represented in the form of "tables of inheritance". The top row of Fig. 2 shows the example of two simplest cases, conventionally referred to as monohybrid and dihybrid cases of a tensor hybridization of vibrosystems. In the first case, the tensor product of two (2*2)-matrices V and W, which have the same set of eigenvalues A and a, gives the (4*4)-matrix $Q=V \otimes W$ with its 4 eigenvalues A*A, A*a, A*a, a*a. In the second case, the tensor product of (4*4) matrices having the same set of eigenvalues AB, Ab, aB, ab, gives (16*16)-matrix with 16 eigenvalues, representated in the tabular form.

		maternal spectrum	
		A	a
paternal spectrum	A	AA	Aa
	a	aA	aa

		maternal spectrum			
		AB	Ab	aB	ab
pat. sp.	AB	AABB	AABb	AaBB	AaBb
	Ab	AABb	AAbb	AaBb	Aabb
	aB	AaBB	AaBb	aaBB	aaBb
	ab	AaBb	Aabb	aaBb	aabb

		maternal gametes	
		A	a
paternal gametes	A	AA	Aa
	a	aA	aa

		maternal gametes			
		AB	Ab	aB	ab
pat. gam.	AB	AABB	AABb	AaBB	AaBb
	Ab	AABb	AAbb	AaBb	Aabb
	aB	AaBB	AaBb	aaBB	aaBb
	ab	AaBb	Aabb	aaBb	aabb

Figure 2: Top row: the tables of inheritance of eigenvalues of (2^n*2^n)-matrices present the result of their tensor product (shown cases of monohybrid and dihybrid tensor hybridizations). Bottom row: examples of Punnet squares for monohybrid and dihybrid hybridizations of organisms under the laws of Mendel. Abbreviations «pat.sp.» and «pat. gam.» mean «paternal spectrum» and «paternal gametes».

The author notes that these tables of inheritance for spectra of vibrosystems are identical to Punnet squares for poly-hybrid crosses of organisms (Fig. 2 below). In genetics from 1906 year, Punnet squares represent Mendel's laws of inheritance of traits under poly-hybrid crosses. (http://en.wikipedia.org/wiki/Punnett_square). Only in Punnet squares, instead of eigenvalues of matrices and their combinations, exist similar combinations of dominant and recessive alleles of genes from parent reproductive cells - gametes.

This formal analogy - between Punnet squares of combinations of alleles and tables of tensor inheritance of eigenvalues of matrices of vibrosystems - generates the following idea:

• alleles of genes and their combinations can be interpreted as eigenvalues of (2^n*2^n)-matrices from tensor families of matrices of oscillatory systems. For genetic systems, this model approach focuses an attention on the possible importance a particular class of mutually related resonances from tensor families of matrices, which play the role of biological "matrix archetypes."

3. GENETIC ALPHABETS AND TENSOR SYSTEMS OF RESONANCES

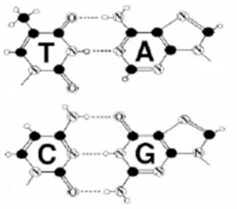
In the past century, science has discovered that molecular bases of genetic coding are identical in all species of organisms. A new understanding of life appeared: *«Life is a partnership between genes and mathematics»* [Stewart, 1999]. All physiological systems of the body should structurally consistent with the genetic code for their reproductions in descendants to avoid extinction; these questions are studied in the field of genetic biomechanics [Petoukhov, 2014]. But what kind of mathematics is a partner of the genetic code, a system of which has noise-immunity properties? Trying to find such mathematics, the author has got some results about connections of the complex system of genetic alphabets with the formalisms of the theory of noise-immunity coding [Petoukhov, 2008a,b, 2011, 2012, 2014]. In this article he puts forward and studies the hypothesis that mathematics of resonances of oscillation systems with many degrees of freedom is one of the partners of genes. It is known that resonances may be used to transmit information (including noise-immunity way), for example, in speech communication. According to the classics of structural linguistics (R.Jakobson et al.), our language did not come out of nowhere, but it is a superstructure over the oldest language - the genetic language (see details in [Petoukhov, He, 2010, Chapter 12]). This is one of the reasons to investigate the genetic system, including genetic alphabets, from the standpoint of mentioned mathematics of resonances. In the framework of the said hypothesis can be assumed that the genetic alphabets are based on systems of resonances, or more precisely on spectra of eigenvalues and eigenvectors of the tensor families of $[2^n*2^n]$-matrices. From this perspective, we represent one of the variants of analysis of features of the genetic alphabets, which testifies in favor of this hypothesis.

As is known, the molecule of heredity DNA contains a sequence of four nitrogenous bases in the role of four "letters" of the basic genetic alphabet of DNA: adenine A, cytosine C, guanine G, thymine T. The alphabet is divided into a pair of complementary letters: A-T and C-G, which stand on two strands of DNA always opposite each other. Genetic code encodes sequences of 20 amino acids in proteins using 64 triplets representing all possible combinations of these four types of the letters: CAG, GCT, ATC, ... The system of genetic coding is based on sets (alphabets) of n-plets: the set of

4 monoplets (nitrogenous bases A, C, G, T); the set of 4^2 = 16 doublets (AA, AC, ...); the set of 4^3 = 64 triplets.

Let us assume that four nitrogenous bases of DNA are eigenvalues of some matrices and so they can be located on diagonals of the corresponding diagonal matrices. In this case the following information is useful: 1) any square matrix with distinct eigenvalues λ_i is transformed into its diagonal form (due to selection of the basis), in which all its eigenvalues lie on its diagonal, and all other terms are equal to zero; 2) the tensor product of diagonal matrices always generates a diagonal matrix.

Science does not know why the basic alphabet of DNA consists of the four poly-atomic structures A, C, G, T of very simple structure. But it is known that the set of these four structures are not quite heterogeneous, but it carries on itself the symmetric system of binary-oppositional traits. The system of such traits divides the genetic four-letter alphabet into various three pairs of letters, which are equivalent from a viewpoint of one of these traits or its absence (Fig. 3): 1) C=T & A=G (according to the binary-opposite traits: "pyrimidine" or "non-pyrimidine", that is purine); 2) A=C & G=T (according to the traits: amino or keto; 3) C=G & A=T (according to the traits: three or two hydrogen bonds are materialized in these complementary pairs) [Petoukhov, 2008a].



TRAITS	G	A	T	C
1) pyrimidine (C,T), purine (A,G)	0_1	0_1	1_1	1_1
2) amino (A,C), keto (G,T)	0_2	1_2	0_2	1_2
3) complementarity (C,G) and (A,T) with 3 or 2 hydrogen bonds	1_3	0_3	0_3	1_3

Figure 3: Three binary sub-alphabets according to three kinds of binary-opposite traits in a set of nitrogenous bases G, A, T, C.

Now in cases of the alphabets of 16 doublets and of 64 triplets, we will continue this natural scheme of the division into sub-alphabets on the base of the principle of pairing letters. Imagine, for example, the amino-pair A and C and the keto-pair G and T in the role of diagonal members of two diagonal (2*2)-matrices, i.e. in the role of their eigenvalues (Fig. 4). Then the set of all variants of tensor products of these two diagonal (2*2)-matrices represents the entire set of 16 doublets in the form of diagonals of the 4 diagonal (4*4)-matrices with an ordered arrangement of doublets. Wherein each of 16 doublets is one of eigenvalues of one of the matrices, defining its corresponding eigenvector (Fig. 4). The index "d" after the brackets we use for short notation of diagonal matrices. Tensor products of the same two diagonal matrices $[C, A]_d$ and [T,

$G]_d$ in all possible combinations in threes represent the entire set of 64 triplets in the form of diagonals of 8 diagonal (8*8)-matrices (the octet of diagonals in Fig. 4 below).

$\begin{vmatrix} C, 0 \\ 0, A \end{vmatrix} = [C, A]_d$; $\begin{vmatrix} T, 0 \\ 0, G \end{vmatrix} = [T, G]_d$	$[C, A]_D \otimes [C, A]_D = [CC, CA, AC, AA]_D$ $[C, A]_D \otimes [T, G]_D = [CT, CG, AT, AG]_D$ $[T, G]_D \otimes [C, A]_D = [TC, TA, GC, GA]_D$ $[T, G]_D \otimes [T, G]_D = [TT, TG, GT, GG]_D$

$[C,A]_d \otimes [C,A]_d \otimes [C,A]_d = [CCC, CCA, CAC, CAA, ACC, ACA, AAC, AAA]_d$
$[C,A]_d \otimes [C,A]_d \otimes [T,G]_d = [CCT, CCG, CAT, CAG, ACT, ACG, AAT, AAG]_d$
$[C,A]_d \otimes [T,G]_d \otimes [C,A]_d = [CTC, CTA, CGC, CGA, ATC, ATA, AGC, AGA]_d$
$[C,A]_d \otimes [T,G]_d \otimes [T,G]_d = [CTT, CTG, CGT, CGG, ATT, ATG, AGT, AGG]_d$
$[T,G]_d \otimes [C,A]_d \otimes [C,A]_d = [TCC, TCA, TAC, TAA, GCC, GCA, GAC, GAA]_d$
$[T,G]_d \otimes [C,A]_d \otimes [T,G]_d = [TCT, TCG, TAT, TAG, GCT, GCG, GAT, GAG]_d$
$[T,G]_d \otimes [T,G]_d \otimes [C,A]_d = [TTC, TTA, TGC, TGA, GTC, GTA, GGC, GGA]_d$
$[T,G]_d \otimes [T,G]_d \otimes [T,G]_d = [TTT, TTG, TGT, TGG, GTT, GTG, GGT, GGG]_d$

Figure 4: Top left: the initial diagonal (2*2)-matrices with pairs C and A, T and G. Above right: 16 doublets in the role of diagonals of the 4 diagonal (4*4)-matrices. Below: 64 triplets in the role of diagonals of the 8 diagonal (8*8)-matrices.

It is known that code values of triplets are dependent on the order of letters in them. For example, triplets AAC, ACA and CAA, which are identical in their letter composition and which belong to the first of octets in Fig. 4, encode different amino acids. In our approach, each of the triplets has its own personality, because it plays the role of one of eigenvalues of one of the above (8*8)-matrices and it belongs to its individual eigenvector, which is one of 8 unit vectors of an appropriate 8-dimensional space). It means that in this model approach three triplets AAC, ACA, CAA are essentially different because each of them is connected with its eigenvector, i.e. with its own form of oscillations inside an oscillatory system with 8 degrees of freedom.

Each of traits of nitrogenous bases A, C, G, T in Fig. 3 can be interpreted as connected with its own resonance characteristics. For example, it is obvious that purines may have resonance characteristics that differ from the resonance characteristics of pyrimidines due to differences in the structure of the purine and pyrimidine molecules. In this light, each of mentioned pairs of binary-oppositional traits can be treated as a pair of binary-oppositional kinds of resonance characteristics. In this case, numeric symbols 0 and 1 in each of binary sub-alphabets in Fig. 5 are representations of binary-oppositional kinds of resonance characteristics. This idea connects physical concepts of resonances of vibrosystems with abstract binary-numeric systems of computer technology and mathematics, including dyadic groups of binary numbers. For comparison, we recall that in computer technology binary elements 0 and 1 are physically realized through using two types of signal amplitudes (eg oppositional in polarity) or two kinds of laser beams, etc., but in the considered genetic case, the binary opposition of the resonance characteristics is supposed that gives an opportunity to consider genetic systems as computers.

4. SYMMETRICAL PROPERTIES OF 8 OCTETS OF TRIPLETS

The insertion into the octets (Fig. 4), which were constructed very formally, those amino acids and stop-codons, which are encoded by triplets, discovers a hidden symmetry in this octet organization: the entire set of 8 octets is divided into four pairs of adjacent octets with the same list of amino acids and stop-codons inside each pair (Fig. 5).

CCC PRO	CCA PRO	CAC HIS	CAA GLN	ACC THR	ACA THR	AAC ASN	AAA LYS
CCT PRO	CCG PRO	CAT HIS	CAG GLN	ACT THR	ACG THR	AAT ASN	AAG LYS
CTC LEU	CTA LEU	CGC ARG	CGA ARG	ATC ILE	ATA MET	AGC SER	AGA STOP
CTT LEU	CTG LEU	CGT ARG	CGG ARG	ATT ILE	ATG MET	AGT SER	AGG STOP
TCC SER	TCA SER	TAC TYR	TAA STOP	GCC ALA	GCA ALA	GAC ASP	GAA GLU
TCT SER	TCG SER	TAT TYR	TAG STOP	GCT ALA	GCG ALA	GAT ASP	GAG GLU
TTC PHE	TTA LEU	TGC CYS	TGA TRP	GTC VAL	GTA VAL	GGC GLY	GGA GLY
TTT PHE	TTG LEU	TGT CYS	TGG TRP	GTT VAL	GTG VAL	GGT GLY	GGG GLY

Figure 5: Neighboring octets of triplets in any of the four pairs 1-2, 3-4, 5-6, 7-8 are identical to each in lists of encoded amino acids and stop-codons (an appropriate amino acid or stop-codon are shown under each triplet for the case of the Vertebrate Mitochondrial Code, which is the most symmetric among known dialects of the genetic code) [Petoukhov, 2014])

Another hidden symmetry in these octets of triplets (Fig. 4) is associated with the phenomenon of natural structurization of the entire set of 64 triplets into two equal subsets by means of traits of strong and weak roots (code values of triplets with strong roots do not depend on the letters on their third position, and code values of triplets with weak roots depend on the third letter [Rumer, 1968]): a) 32 triplets with strong roots, ie with 8 "strong" doublets AC, CC, CG, CT, GC, GG, GT, TC on their first positions; b) 32 triplets with weak roots, ie with 8 "weak" doublets AA, AG, AT, GA, TA, TC, TG.Whether there is symmetry in the arrangement of the triplets with strong and weak roots inside the 8 octets of triplets that were built formally without mentioning amino acids? Note that the huge quantity 64! ≈ 10^{89} of variants exists for dispositions of 64 triplets in 8 octets, ie in 64 cells. For a comparison, the modern physics estimates a duration of existence of the Universe in 10^{17} seconds. Obviously, the random arrangement of 20 amino acids and of corresponding triplets in 64 cells is almost never give symmetry in such set of octets. But unexpectedly the phenomenological location of 32 triplets with strong roots (black color) and 32 triplets with weak roots (white color) has the symmetric character in these octets (Fig. 6, left): 1) the black-and-white mosaic of each octet is mirror-antisymmetric in its left and right halves and it has a meander-like character; 2) the whole set of 8 octets is divided into a pair of adjacent octets with identity mosaics. But such odd meander functions are well known in signal processing theory and probability theory under the name "Rademacher functions": $r_n(x) = \text{sign}(\sin 2^n \pi x)$ (http://www.encyclopediaofmath.org/index.php/Rademacher_system). Rademacher functions, containing only the elements "+1" and "-1", are phenomenologically associated with the genetic alphabet: each of the 8 octets of triplets is one of Rademacher functions if every black (white) triplet interpreted as an element "+1" ("-1").

[CCC, CCA, CAC, CAA,	ACC, ACA, AAC, AAA]$_d$
[CCT, CCG, CAT, CAG,	ACT, ACG, AAT, AAG]$_d$
[CTC, CTA, CGC, CGA,	ATC, ATA, AGC, AGA]$_d$
[CTT, CTG, GGT, CGG,	ATT, ATG, AGT, AGG]$_d$
[TCC, TCA, TAC, TAA,	GCC, GCA, GAC,GAA]$_d$
[TCT, TCG, TAT, TAG,	GCT, GCG, GAT, GAG]$_d$
[TTC, TTA, TGC, TGA,	GTC, GTA, GGC, GGA]$_d$
[TTT, TTG, TGT, TGG,	GTT, GTG, GGT, GGG]$_d$

[CCC, CCA, CAC, CAA, ACC, ACA, AAC, AAA]$_D$
[CCT, CCG, CAT, CAG, ACT, ACG, AAT, AAG]$_D$
[CTC, CTA, CGC, CGA, ATC, ATA, AGC, AGA]$_D$
[CTT, CTG, GGT, CGG, ATT, ATG, AGT, AGG]$_D$
[TCC, TCA, TAC, TAA, GCC, GCA, GAC, GAA]$_D$
[TCT, TCG, TAT, TAG, GCT, GCG, GAT, GAG]$_D$
[TTC, TTA, TGC, TGA, GTC, GTA, GGC, GGA]$_D$
[TTT, TTG, TGT, TGG, GTT, GTG, GGT, GGG]$_D$

Figure 6. Left: the symmetric location of triplets with strong and weak roots in eight octets of triplets is characterized by the symmetric mosaic of Rademacher functions. Right: the location of the triplets, which differ by the special status of thymine T in them, is characterized by the mosaic of the complete orthogonal system of Walsh functions. Explanation in the text.

Another interesting structural feature of the 8 octets of triplets is connected with the phenomenon of the special status of the T (thymine) in the basic alphabet of DNA. Among the four DNA bases - A, C, G, T - the letter T contrasts phenomenologically with three other letters of the alphabet: 1) only the letter T is transformed into another letter U (uracil) in the transition from DNA to RNA; 2) only the letter T (and its changer U) has not the functionally important aminogroup NH2 in contrast of other three letters (see Fig. 3, left side). This binary opposition can be expressed in a digital form as: A = C = G = + 1 and T = -1. Then each triplet under replacing its letters on these numbers (A = C = G = + 1, T = -1) can be represented as the product of these numbers. For example, the triplet CAT is represented as 1*1*(-1) = -1 and the triplet TGT - as (-1)*1 *(-1) = +1. In the result, the 8 octets of triplets obtain numerical representations as sequences of elements +1 and -1 (these elements are correspondingly marked by black and white colors on Fig. 6, right side). The set of these sequences coincide with the complete system of orthogonal Walsh functions for 8-dimensional spaces.

These Walsh functions, containing only the elements of "+1" and "-1", are widely used in digital signal processing and noise-immunity coding (http://en.wikipedia.org/wiki/Walsh_function). Completeness of the system of 8 Walsh functions means that any 8-dimensional vector can be represented as their superposition (ie decomposed on their base). On the base of complete systems of Walsh functions, noise-immunity coding of information is used on the spacecrafts "Mariner" and "Voyager" for transmission to Earth photos of Mars, Jupiter, Saturn, Uranus and Neptune. Complete systems of Walsh functions form Hadamard matrices used in quantum computers ("Hadamard gates"); Hadamard matrices are used in quantum mechanics in the form of unitary operators, etc. [Ahmed, Rao, 1975; Harmuth, 1970, 1977, 1981; Seberry, Wysocki, Wysocki, 2005]. In our approach, these systems of Walsh functions are representatives of the genetic alphabet: each of the 8 functions of the complete Walsh system is the diagonal of one of the genetic (8*8)-matrices of the diagonal type, ie a spectrum of eigenvalues of an oscillation system with 8 degrees of freedom.

These results testify in favor of the following: the alphabets of the genetic code are alphabets of eigenvalues and eigenvectors of matrices of oscillatory systems (figuratively speaking, the genetic code is the code of resonances); respectively, genetic texts based on these alphabets can be considered as texts written in the language of the resonances. No wonder that any genetically inherited organism is a chorus of agreed oscillatory processes. Here one can recall about double Nobel laureate Linus Pauling, who proposed in 1928 the theory of resonances for the electronic structure of molecules and also the idea of hybridization of atomic orbitals (http://en.wikipedia.org/wiki/Linus_Pauling). Possible theoretic and applied studies of genetic systems of resonant frequencies can be connected by the general name: "the conception of resonant genomes".

5. ON BASIC FREQUENCIES OF COLOR PERCEPTION

Let's apply the table of tensor inheritance of eigenvalues of matrices to our genetically inherited color perception. According to the three-component theory of color vision, it is based on three basic colors - red, green and blue. Photosynthesis, which is much older than color vision, uses red and blue colors, since plant leaves reflect green color, whereby the leaves look green. Therefore, one can assume that red and blue colors are biologically more "basic" than the green.

Let's go back to the table of tensor inheritance of eigenvalues in Fig. 2 in the case of the monohybrid cross of (2*2)-matrices of two "parental" vibrosystems with eigenvalues (A, a) and natural frequencies $\Omega_1=A^{0.5}$ and $\Omega_2=a^{0.5}$. Fig. 7 shows the corresponding table of inheritance of squares of natural frequencies. This table shows that there are only three kinds of natural frequencies in the generated (4*4)-matrix: W_1, W_4 and lying between them the frequency $W_2 = W_3 = \Omega_1^2 * \Omega_2^2$, which twice repeated in the table (denote this frequency as $W_{2,3}$). This table reveals the following relation among these three frequencies: $W_{2,3} = (W_1*W_4)^{0.5}$.

		SPECTRUM W	
		Ω_1^2	Ω_2^2
SPECTRUM V	Ω_1^2	$W_1^2 = \Omega_1^4$	$W_2^2 = \Omega_1^2*\Omega_2^2$
	Ω_2^2	$W_3^2 = \Omega_1^2*\Omega_2^2$	$W_4^2 = \Omega_2^4$

Figure 7: The table of inheritance of squares of natural frequencies in the monohybrid case of the tensor hybridization of (2*2)-matrices

As is known, the range of perception of red color is 430-480 THz and blue color - 610-670 THz (http://en.wikipedia.org/wiki/Color). Let's take the median frequency 455 THz of the range of red color on the role of the frequency W_1, and the median frequency 640 THz of the range of blue color on the role of the frequency W_4. Then the relation $W_{2,3} = (W_1*W_4)^{0.5} = (455*640)^{0.5}$ gives the value 539.6 THz of the third natural frequency of the oscillatory system. But this value of the frequency $W_{2,3}$, which is repeated twice in the table of inheritance (Fig. 7), coincides with the known value of the frequency 540 THz of maximum sensitivity of our vision attributable to green color.

Consequently, our color perception is one of biological examples that allow their modeling on the base of tensor-spectral approach presented in this paper. And its natural features can be treated in connection with the Mendel laws and natural frequencies of tensor-hybrid vibrosystems. This gives a new approach to the relationship between music and color harmonies, that was noted else by Newton, as well as to color therapy, color music, etc.

6. ON THE WEBER-FECHNER LAW

Let's turn to the main psychophysiologic law by Weber-Fechner (http://en.wikipedia.org/wiki/Weber–Fechner_law): the intensity of the perception is proportional to the logarithm of stimulus intensity; it is expressed by the equation

$$p = k*\ln(x/x_0) = k*\{\ln(x) - \ln(x_0)\}, \qquad (6)$$

where p - the intensity of perception, x – stimulus intensity, x_0 - threshold stimulus, ln – natural logarithm, k – a weight factor. It is known that different types of inherited sensory perception are subordinated to this law: sight, hearing, smell, touch, taste, etc. Because of this law, the power of sound in music and technology is measured on a logarithmic scale in decibels.

One can suppose that the innate Weber–Fechner law (WF-law) is the law especially for nervous system. But it is not so since its meaning is much wider because it holds true in many kinds of lower organisms without a nervous system in them: "*this law is applicable to chemo-tropical, helio-tropical and geo-tropical movements of bacteria, fungi and antherozoids of ferns, mosses and phanerogams ... . The Weber-Fechner law, therefore, is not the law of the nervous system and its centers, but the law of protoplasm in general and its ability to respond to stimuli*" [Shultz, 1916, p. 126].

The conception of resonant genomes reveals that this logarithmic WF-law (6) is simply modelled on the base of natural resonant frequences of a particular class of vibrational systems with 2 degrees of freedom. Really, one can note that the natural logarithm in the WF-law (6) is deeply connected with the hyperbola y=1/x: the natural logarithm can be defined for any positive real number a as the area under the curve y = 1/x from 1 to a (Fig. 8). It was formerly also called hyperbolic logarithm, as it corresponds to the area under a hyperbola (http://ctu.edu.vn/~dvxe/econometrics/Natural_logarithm.pdf). History of hyperbolic logarithms is described for example in the book [Klein, 2004].

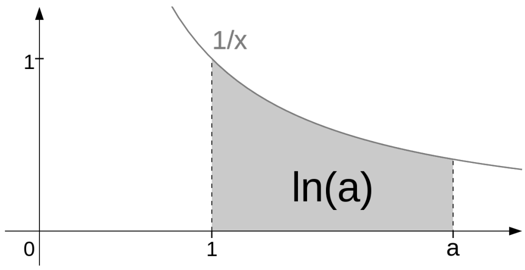


Figure 8: ln(a) illustrated as the area under the curve f(x) = 1/x from 1 to a.

The proposed model approach considers two related oscillatory biosystems with 2 degrees of freedom in each. They are represented by diagonal matrices [x_0 0; 0 y_0] and [x 0; 0 y] with eigenvalues, which are related reciprocally: $y_0=1/x_0$ and y=1/x (correspondingly, natural resonant frequences in every of the systems are also reciprocal to each other). The pair of eigenvalues of each of the matrices defines its own point on the same hyperbola y=1/x and its own meaning of natural logarithm. More precisely, $\ln(x_0)$ is defined by the eigenvalues of the first matrix and ln(x) is defined by the eigenvalues of the second matrix. Subtraction $\ln(x)-\ln(x_0)$ gives the intensity of perception p in the expression (6) of the WF-law, where only a weight factor k should be included (which is different for different sensory channels). Here the eigenvalue x_0 from the first oscillatory system is interpreted as the threshold stimulus; the eigenvalue x from the second oscillatory system is interpreted as a stimulus intensity (that is an input signal). Taking into account that the input signal is changed in time, one should suppose that the second oscillatory biosystem has a property of changing its natural resonant frequency $w=x^{0.5}$ in accordance with changes of input signals for their registration in the sensory channel. One can add that the two oscillatory systems with 2 degrees of freedom in each can be combined in a general oscillatory system with 4 degrees of freedom, which is represented by means of the diagonal (4*4)-matrix $[x_0, x, 1/x, 1/x_0]_d$.

7. CONCLUDING REMARKS

Our concept of inherited systems of biological resonances additionally pays attention to a possible biological significance of phenomena of vibrational mechanics, which is widely used in engineer technologies. This field of mechanics has amazing phenomena of the vibrational separation and structurization of multiphase media, vibro-transportations, vibro-transmitting energy from one part of the system to another, etc. [Blekhman, 2000]. Familiarity with some of these phenomena creates the impression of staying in the world with other physical laws. Practically invisible vibrations, which may have high frequency and small amplitudes, can provide, for example, the following phenomena: a) the upper position of the inverted pendulum becomes stable; b) a heavy metal ball "pops" in a layer of sand; c) rotation of the rotor of the electro-motor, which is disconnected from the power supply, can be steadily supported by means of vibro-transfer of energy from another motor, which is connected to the power supply and which stands on a general vibro-platform with the first motor; d) movements of metronomes, which stand on a general movable platform, become synchronized, etc.In living organisms, numerous phenomena exist, which are associated with the vibrations at different levels and which occur as if they were produced by means of mysterious forces. We hope that appropriate mathematical models of such biological processes will be developed on the base of formalisms of vibrational mechanics and the theory of inherited systems of biological resonances.

Here one can remind also about the known phenomenon of conformational oscillations of enzyme macromolecules, including at frequencies of sound waves. In connection with this phenomenon, S. Shnoll [1979, p.75] wrote about the likely importance for life "*a fantastic picture of musical interactions of biochemical systems, cells, organs*," which belongs to the field of "*biochemical aesthetics*." Our conception of resonant genomes is associated with "biochemical aesthetics". In our model approach, organisms can be considered as music synthesizers with a great number of inherited tunings of resonant modes. Partly in this connection, a special "Center for Interdisciplinary Studies of musical creativity" has been created in the Moscow P. I. Tchaikovsky Conservatory, and also a new class of musical instruments on the basis of the so-called "genetic musical scales" has been patented with the author's participation.

This Center develops intensively a new direction of musical culture on the base of a special set of genetic musical scales, where Fibonacci numbers and the golden section are participated. These mathematical and musical scales have been received in the Russian Academy of Sciences (RAS) in the result of mathematical analysis of harmonical systems of molecular parameters of the genetic code; this study was connected with a many-year cooperation between RAS and the Hungarian Academy of Sciences on the theme "Non-linear models and symmetrologic analysis in biomechanics, bioinformatics and the theory of self-organizing systems" [Petoukhov, 2008a; Darvas, et al., 2012]. A team of professional composers, physicists and computer experts of the Moscow P. I. Tchaikovsky Conservatory and the Russian Academy of Sciences creates effective computer programs and devices for composers and other musicians, who wish to work in the field of genetic music. The first epoch-making concert of genetic music was performed by representatives of the Moscow conservatory in Vienna on 4 June 2015. The concert included the following music: “Five Genetic Miniatures” and a genetic etude “The Hot Sea”, which were performed by their author I.Soshinsky, a member of the Russian Union of Composers; some classical musical pieces for piano were performed in the genetic scales by A. Koblyakov, dean of the Composer faculty of the Moscow conservatory. The concert received very positive reaction of publics. In the result the team of the Moscow Conservatory has been invited to make a new concert of genetic music again in Vienna on summer 2016 during the “Symmetry Festival-2016”. Some other interesting invitations were also received about new concerts of genetic music in different countries, including musical performances with holographic laser show. The theme of genetic music is connected with the described conception of resonances in genetics.

Interestingly, that the development of world science leads to the discovery of new connections between aesthetics and fundamental physics. For example, the realization of the famous golden ratio 1.618... in a natural system of resonances was recently discovered in the quantum-mechanical world [Coldea et al., 2010]. It is appropriate to recall the dictum of B.Bolzano: "Cognition is the searching of analogies".

Taking into account the opportunities of vibro-transfers of energy by means of resonant interactions, living organisms can be considered as a resonant consumer of energy of surrounding electromagnetic waves coming from cosmos and depths of the earth. Photosynthesis, which is carried out due to the absorption of energy of solar light waves, is probably only one of examples of biological consumption of external energy on the basis of resonance approvals. Accordingly, any organism should be considered as a part of the global ensemble of wave processes (primarily electromagnetic), waiting for a discovery of new mechanisms of its resonance interactions with external wave world. The desribed genetic conception supposes that living organisms differ from inanimate objects by inherited tensor-matrix coordinations of systems of resonances in them.

Our results of the tensor-matrix modeling testify in favor of the following:

- alphabets of the genetic code are alphabets of resonances; respectively, the genetic code is the code of special systems of resonances, and genetic texts on the basis of these alphabets are texts written in the language of these resonances;

- Punnet squares, describing in genetics poly-hybrid crosses under the laws of Mendel, are analogies of the introduced tables of tensor inheritance of eigenvalues in cases of tensor multiplications of (2*2)-matrices;

- alleles of genes, that appear in accordance with Mendel's laws, can be interpreted as resonances (the eigenvalues) of some oscillatory systems;

- binary-oppositional properties of the DNA alphabet, which are treated as some oppositions of resonance characteristics, determine its binary sub-alphabets and provide a possibility of binary-numeric representations of the alphabets and n-plets of DNA to study the living body as a computer;

- heritable biological processes are associated with the phenomena of vibrational mechanics.

In the past century, science has discovered that molecular-genetic bases of all living organisms are the same (alphabets of DNA, RNA, etc.) and that they are very simple. A hope arises that the algorithmic foundations of organisms, which are subordinated to genetic laws such as Mendel's laws, are also very simple and are unified for all living things. Identifying these algorithms of living matter is important. We assume that the algorithms of resonant matching and ordering subsystems, which are associated with formalisms of tensor products of matrices, play one of key roles in living matter. It seems that study of symmetries will be very important in these researches [Darvas, 2015].

A limited volume of the article allows submitting here only fragments of our research on the genetic theory of bio-resonance systems. For example, as known, the eigenvalues of self-adjoint matrices exist in the foundations of quantum mechanics important for genetic molecules; but an analysis of a possible genetic role of this fact is beyond the scope of this article. Our study has a number of practical aspects related to using of wave and vibration processes in medicine, biotechnology, artificial intelligence systems, etc. In particular it concerns a problem of "genetic music" [Darvas, et al., 2012].

The author believes that the development of modern theoretical biology - as a branch of mathematical natural science - can go on the same way as the development of modern theoretical physics, which, according to P. Dirac, should be by the following recipe. "*Start with a beautiful mathematical theory. "If it is really beautiful* - he belived - *it is sure to be an excellent model of important physical phenomena. So you need to search for these phenomena to develop applications of beautiful mathematical theory and interpret them as predictions of new laws of physics "- in such way, according to Dirac, the whole new physics is built - relativistic and quantum"* (quote from [Arnold, 2006]). In this article it is shown that beautiful mathematical theory of eigenvalues and eigenvectors of the tensor families of matrices gives the model of important genetic phenomena and structures with revealing their deep connection with the theory of resonances of oscillation systems with many degrees of freedom.

REFERENCES

Ahmed, N., Rao, K. (1975) *Orthogonal transforms for digital signal processing*. N.-Y.: Springer-Verlag Inc.

Arnold, V.I. (2006) Publichnaya lektsiia v Matematicheskom institute imeni V.A. Steklova [in Russian, "Public lecture at Mathematical Institute by V.A. Steklov", http://elementy.ru/lib/430178/430281

Balonin N.A. (2000) *Novyi kurs teorii upravleniia dvizheniem* [in Russian, "New course on the theory of *motion control*"], Saint Petersburg: Saint Petersburg State University, 160 p.

Bellman R. (1960) *Introduction to Matrix Analysis*. New York: MacGraw-Hill Book Company.

Blekhman I.I. (2000) *Vibrational mechanics*. Singapore: World Scientific, 509 p.

Coldea R., D. A. Tennant, E. M. Wheeler, E. Wawrzynska, D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl, K. Kiefer (2010). Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E8 Symmetry. *Science*, Jan. 8, p. 177-180.

Darvas G., Koblyakov A., Petoukhov S., Stepanyan I. (2012) Symmetries in molecular-genetic systems and musical harmony. *Symmetry: Culture and Science*, 23, № 3-4, 343-375.

Darvas G. (2015) The unreasonable effectiveness of symmetry in the sciences. *Symmetry: Culture and Science,* 26, №1, 39-82.

Gazale M.J. *Gnomon*. *From pharaohs to fractals*. New Jersey: Princeton University Press. 1999. 280 p.

Gladwell G.M.L. *Inverse problems in Vibration*. London: Kluwer Academic Publishers. 2004, 452 p.

Harmuth H. F. (1970) Transmission of information by orthogonal functions. Berlin: Springer.

Harmuth, H. F. (1977) *Sequency theory*. *Foundations and applications*. N.-Y.: Academic Press.

Harmuth H. F. (1981) *Nonsinusoidal waves for radar and radio communication*. N.-Y.: Academic Press.

Klein F. (2004) *Elementary Mathematics from an Advanced Standpoint*: *Arithmetic, Algebra, Analysis*. London: Dover Publications.

Petoukhov, S. V. (2008a) *Matrichnaya genetika, algebry geneticheskogo koda, pomekhoustoychivost'* [in Russian, „*Matrix Genetics, Algebras of the Genetic Code, Noise Immunity*"], Russian Academy of Sciences, Moscow: Publishing House „Regular and Chaos Dynamics", 315 p. (http://petoukhov.com/)

Petoukhov, S. V. (2008b) The degeneracy of the genetic code and Hadamard matrices, arXiv:0802.3366.

Petoukhov S.V. (2011) Matrix genetics and algebraic properties of the multi-level system of genetic alphabets. - *Neuroquantology,* 2011, 9, No 4, p. 60-81.

Petoukhov S.V. (2012) Symmetries of the genetic code, hypercomplex numbers and genetic matrices with internal complementarities. *Symmetry: Culture and Science*, 23, № 3-4, 275-301.

Petoukhov S.V. (2014) Matrix genetics: algebra of projection operators, cyclic groups and inherited ensembles of biological cycles. In: *Proceedings of 2nd International Conference "Theoretical Approaches to BioInformation Systems*". Eds. B. Dragovich, R. Panajotović, D. Timotijević. Conference organized by Institute of Physics, Serbia (September 17 – 22, 2013, Belgrade, Serbia). Belgrade: Institute of Physics, 189-204, http://www.tabis2013.ipb.ac.rs/tabis2013.pdf

Petoukhov, S.V., He M. (2010) *Symmetrical Analysis Techniques for Genetic Systems and Bioinformatics: Advanced Patterns and Applications*. Hershey, USA: IGI Global, 271 p.

Rumer, Yu.B. (1968) Sistematizatsiia kodonov v geneticheskom kode [in Russian, "Systematization of the codons in the genetic code]. *Doklady Akademii Nauk SSSR, 183*(1), 225-226.

Seberry J., Wysocki B.J., Wysocki T.A. (2005) On some applications of Hadamard matrices. *Metrica*, 62: 221-239.

Shnoll, S. (1979) *Fiziko-himicheskie factory biologicheskoy evolutsii* [in Russian, "*Physico-chemical factors of biological evolution"*]. Moscow: Nauka, 263 p.

Shults E. (1916) The organism as a creativity. - In the book "*Questions of theory and psychology of creativity*", Russia, Kharkov, vol.7, p. 108-190 (in Russian, "*Voprosy teorii i psikhologii tvorchestva*").

Stewart I. (1999) *Life's other secret: The new mathematics of the living world*. N.-Y.: Penguin.

Weinberger, N.M. (2004) Music and brain. *Scientific American, 291*(5), 88-95.