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Modeling inherited physiological structures based on hyperbolic numbers

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ABSTRACT

The article is devoted to using the mathematics of multi-dimensional hyperbolic numbers and their matrix representations for modeling of different inherited biosystems, which are parts of the holistic body. The list of considered models includes phyllotaxis sequences; Punnet squares of the Mendelian genetics; the main psychophysical Weber-Fechner law, and some others. This modeling approach reveals hidden structural interconnections among different biosystems and leads a deeper understanding of their commonality related to the commonality of their genetic basis. At the same time, it shows the structural connection of inherited biosystems with formalisms of the theory of resonances of oscillatory systems with many degrees of freedom. This complements works of many authors - from antiquity to the present day - on the importance of coordinated oscillations and resonances in the life of living organisms. The modeling approach based on hyperbolic numbers and their bisymmetric matrices reveals that the structural commonality of different inherited biosystems is related to the harmonic progression $1/n$ and the harmonic mean, which are long known in arts and culture. They are used in the Pythagorean teaching on the musical harmony and the aesthetics of proportions. Besides, the using - for biosystems modeling - of multi-dimensional hypercomplex numbers, which are one of the main tools in modern mathematical natural sciences, opens a bridge between these sciences and biology for their mutual enrichment.

1. Introduction

Living organisms are informational entities, in which everything is subordinate to the task of reliably transmitting genetic information to descendants. All physiological systems must be argued with a genetic coding system to be genetically encoded for their survival and inheritance into the next generations. In this regard, the structures of the molecular genetic system play the role of a fundamental "tuning fork" for matching various inherited physiological structures combined into a single organism and working in concert. Correspondingly, the structural organization of physiological systems can bear the imprint of the structural features of molecular genetic systems.

This article continues previous author's publications (Petoukhov, 2020a,b) on structural connections of the genetic coding system with bisymmetric matrices presenting hyperbolic numbers $a+b\mathrm{j}$ (where j is the imaginary unit of hyperbolic numbers with its property $\mathrm{j}^2 = +1$). These numbers and their algebraic extensions are one of the well-known types of multidimensional numbers and are related - via their representation by bisymmetric matrices – to formalisms of the theory of resonances of oscillatory systems with many degrees of freedom. Analysis of the molecular system of binary-oppositional features of four DNA nucleotides (adenine A, thymine T, cytosine C, and guanine G) revealed the connection of DNA-alphabets of 4 nucleotides, 16 doublets, and 64 triplets with the tensor family of bisymmetric matrices $[C, A; G, T]^{(n)}$. The mentioned alphabetical elements of DNA are located in the cells of these matrices in a strictly ordered manner. The features of the family of these genetic matrices led the author to develop a new method for analyzing the oligomeric composition of long DNA sequences, called the method of oligomeric sums. Application of this method to the analysis of long nucleotide DNA sequences, which are considered as multilayer texts from oligomers of different lengths, revealed the existence of hyperbolic rules of oligomeric cooperative organization of DNA sequences in all tested genomes of eukaryotes and prokaryotes (Petoukhov, 2020a, b). These universal rules of genomes turned out to be associated with the harmonic progression $1/1, 1/2, \ldots, 1/n$, long known in mathematics and physics thanks to the works of Pythagoras, Orem, Leibniz, Newton, Euler, Fourier, Dirichlet, Riemann, and other researchers. Its historical name - the harmonic progression - was given because of its connection with a number of harmonics in music and the lengths of standing waves in an oscillating string. This new article continues the analysis of connections of genetic structures with the harmonic progression and with bisymmetric matrices, which represent hyperbolic numbers and are related to the theory of resonances.

Mathematical features of hyperbolic numbers $a+b\mathrm{j}$ and their

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representations by bisymmetric matrices [*a*, *b*; *b*, *a*] were described early in (Petoukhov, 2008; Petoukhov and He, 2010). On this algebraic modeling approach, one can hope to better understand the deep structural connection of inherited physiological systems with the genetic coding system and the theory of resonances. Also, new possibilities of algebraic modeling of biological phenomena based on multidimensional numerical systems arise and are developed concerning the known point of view: "*life is a partnership between genes and mathematics*" (Stewart, 1999).

As known, different branches of modern science use various kinds of multi-dimensional numbers: complex numbers, double numbers, dual numbers, quaternions, and other hypercomplex numbers. These multi-dimensional numeric systems play the role of the magic tool for the development of theories and calculations in problems of heat, light, sounds, fluctuations, elasticity, gravitation, magnetism, electricity, current of liquids, quantum-mechanical phenomena, special theory of relativity, nuclear physics, etc. It seems important to investigate more deeply the possibilities of using of mentioned hyperbolic numbers in algebraic modeling the structural interrelations between the molecular-genetic system and inherited physiological systems.

Gr. Mendel found that the transmission of traits during the crossing of organisms occurs by certain algebraic rules, despite the colossal heterogeneity of molecular structures of their bodies. According to Mendel's law of independent inheritance of traits, information from micro-world of genetic molecules dictates macrostructures of living organisms, despite strong noise and interference, through many independent channels (for instance, colors of hair, eye, and skin are inherited independently from each other). This determinism is carried out using unknown algorithms of multi-channel noise-immunity coding. Consequently, every organism is an algorithmic machine of multi-channel noise-immunity coding.

In a broad sense, code is usually understood as correspondence between two sets of characters. For example, a usual phone book can be considered as a coding system, in which its phone numbers encode names of people. But for algorithmic providing a noise-immunity transfer of information, modern communication technologies widely use more complex kinds of codes termed as algebraic codes and algebra-geometric codes. Correspondingly, one can believe that the genetic code having noise-immunity is akin to these algebraic noise-immunity codes based on formalisms of mathematical matrices.

2-dimensional hyperbolic numbers and their 2^n-dimensional algebraic extensions, based on the tensor product of bisymmetric matrices, have their representation in forms of bisymmetric matrices related, in particular, with formalisms of the theory of resonances in oscillatory systems with many degrees of freedom. Below these well-known mathematical peculiarities of hyperbolic numbers with their matrix representation and also their use for modeling some inherited physiological structures are described. The article (Petoukhov, 2016) describes deep structural connections of molecular alphabets of the genetic code with eigenvalues and eigenvectors of symmetric matrices of oscillatory systems: figuratively speaking, according to the described model approach in the conception of multi-resonance genetics, the genetic code is the code of systems of resonances. This new article substantiates and analyzes the possibilities of modeling inherited supramolecular biostructures in connection with that part of the theory of resonances, which is based on bisymmetric matrices representing hyperbolic numbers and special oscillatory systems with many degrees of freedom.

One should recall that matrices possess a wonderful property to express resonances, which sometimes is called as their main quality (Balonin, 2000, p. 21, 26; Bellman, 1960). The physical resonance phenomenon is familiar to everyone. The expression $y = A^*S$ models the transmission of a signal S via an acoustic system A, represented by a relevant matrix A. If an input signal is a resonant tone, then the output signal will repeat it with a precision up to a scale factor $y = \lambda^*S$ by analogy with a situation when a musical string sounds in unison with the neighboring vibrating string. In the case of a matrix A, its number of resonant tones S_i corresponds to its size. They are called its eigenvectors, and the scale real factors λ_i with them are called its eigenvalues or, briefly, spectrum A. Frequencies $\omega_i = \lambda_i^{0.5}$ (Gladwell, 2004, p. 61) are defined as natural frequencies of the system, and the corresponding eigenvectors are defined as its own forms of oscillations (or simply, natural oscillations). This article will pay special attention to the eigenvalues of bisymmetric matrices modeling the biostructures under consideration. Such matrices have real eigenvalues and their eigenvectors are orthogonal. Bisymmetric matrices correspond special types of oscillatory systems with many degrees of freedom and appropriate sets of resonances (for comparison, complex numbers are represented by asymmetric matrices whose eigenvalues are complex numbers not related to the expression of resonances).

Among many works about resonances in different systems, the theory of resonances by Linus Pauling in structural chemistry takes an important place (Pauling, 1940). His theory uses the fundamental principle of minimal energy because - in the resonant combining of parts into a single ensemble - each of members of the ensemble requires less energy for performing their own work than when working individually. Pauling claimed that living organisms are chemical in nature, and resonances in their molecules should be very essential for biological phenomena. Mechanisms of resonances can coordinate works of many parts for a cooperative functioning of the holistic system with its energy optimization. The works of many modern authors consider the important role of resonances in the organization of biosystems (for example (Del Giudicel et al., 2015; Ji, 2012, 2018; Marchettini et al., 2010; Montagnier et al., 2015; Pollack, 2013; Reid, 2019; Shushardzhan and Petoukhov, 2020)).

It should be noted that many authors devoted their works to algebraic modeling physiological structures based on multi-dimensional numbers but without any relation to the structures of the genetic code and to the theory of resonances (for example (Bodnar, 1992, 1994; Khrennikov and Asano, 2020; Jean, 1994; Kienle, 1964; Labunets et al., 2002; Lubeburg, 1950; Penrose, 1996; Rapoport, 2016a,b,c; Smolyaninov, 1984, 2000; Stakhov, 2009)). A distinctive feature of the approach described in this article is its focus on identifying the relationships of inherited supramolecular structures with the algebraic features of the genetic coding system associated with hyperbolic numbers and the theory of resonances. This approach allows representing a set of genetic and many other physiological systems of the living body in their hidden interconnections as the whole entity, which is inherited jointly with these interconnections from one to the next generations.

2. Phyllotaxis laws and hyperbolic numbers

In biology, inherited helical morphologic lattices, associated with Fibonacci numbers F_n (Table 1), are studied under the title "phyllotaxis laws" for over 150 years. Such phyllotaxis structures are realized at very different levels and branches of biological evolution. Hundreds of publications are devoted to them (see the review in (Jean, 1994)). Fibonacci numbers are strongly related to the golden ratio $\varphi = (1+5^{0.5})/2 = 1.618$ ...: the ratios F_{n+1}/F_n tend to φ as n increases.

For example, the numbers of left and right spirals in the heads of the sunflower are equal to the neighboring members of the Fibonacci series F_n and F_{n+1}. Similar phyllotaxis structures are found in shoots of plants and trees, scales of coniferous cones and pineapples, and so on, and they are called parastichies. To characterize phyllotaxis of such objects, usually indicate two parameters: number of left spirals and number of right spirals, which are observed on the surface of phyllotaxis objects. Phyllotaxis of structures with such patterns is described by ratios $F_{n+1}/$

Table 1
The Fibonacci sequence of real numbers where $F_{n+2} = F_n + F_{n+1}$.

n	1	2	3	4	5	6	7	8	9	10	…
F_n	1	1	2	3	5	8	13	21	34	55	…

F_n, which denote the order of symmetry of phyllotaxis lattices (1):

$$F_{n+1}/F_n: 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, \dots \quad (1)$$

The sequence of real numbers (1) is termed the "parastichic sequence" (Jean, 1994). The ratios F_{n+1}/F_n denote the order of symmetry of phyllotaxis lattices. In the process of growth of some organisms, their phyllotaxis lattices are transformed with the transition to other phyllotaxis orders of symmetry F_{k+1}/F_k (2):

$$(F_{n+1}/F_n) \to (F_{n+2}/F_{n+1}): 2/1 \to 3/2 \to 5/3 \to 8/5 \to 13/8 \to 21/13 \to \dots \quad (2)$$

The works (Bodnar, 1992, 1994) revealed that such ontogenetic transformations of phyllotaxis lattices correspond to hyperbolic rotations known in the special theory of relativity as Lorentz transformations. On this basis, these works declared that living matter is structurally related to Minkowski geometry. Correspondingly it means that the inherited phyllotaxis phenomena are related to hyperbolic numbers.

Let us deeper analyze connections of phyllotaxis laws with hyperbolic numbers and their bisymmetric matrices. Taking into account that two members F_n and F_{n+1} of the Fibonacci series always exist at once in the genetically inherited systems of parastichies, one can represent these interconnected real members in the form of two parts of a single 2-dimensional hyperbolic number $F_n + j^*F_{n+1}$ represented by its bisymmetric matrix $[F_n, F_{n+1}; F_{n+1}, F_n]$. This proposed representation gives the following two additive series of hyperbolic numbers and their bisymmetric matrices for a new modeling approach for phyllotaxis phenomena and their ontogenetic transformations:

$$F_{n+1} + jF_n: 2 + j,\ 3 + j2,\ 5 + 3j,\ 8 + 5j,\ 13 + 8j,\ 21 + 13j,\ 34 + 21j, \quad (3)$$

$$\begin{vmatrix} F_{n+1}, F_n \\ F_n, F_{n+1} \end{vmatrix} : \begin{vmatrix} 2,1 \\ 1,2 \end{vmatrix}, \begin{vmatrix} 3,2 \\ 2,3 \end{vmatrix}, \begin{vmatrix} 5,3 \\ 3,5 \end{vmatrix}, \begin{vmatrix} 8,5 \\ 5,8 \end{vmatrix}, \begin{vmatrix} 13,8 \\ 8,13 \end{vmatrix} \quad (4)$$

The additive series (3) and (4) are called parastichic sequences of hyperbolic numbers with Fibonacci coordinates. What is the meaning and advantages of the proposed consideration of phyllotaxis phenomena based on such hyperbolic numbers and their matrix representations? Firstly, cardinally new mathematical personages arise in modeling phyllotaxis phenomena: matrix operators of the bisymmetric type; their eigenvectors, which are related to the theory of resonances where they determine the resonant frequencies of corresponding oscillatory systems; orthogonal systems of their eigenvectors. Secondly, analogies arise between phyllotaxis and molecular-genetic structures related to hyperbolic numbers. Thirdly, analogies arise between these biological phenomena and mathematical theories in physics and informatics, including the theory of resonances, where hyperbolic numbers and matrix operators used. Here one can recall that the general idea that knowledge is a search for analogies is recognized in science at least since the time of B. Bolzano.

In the proposed approach, to define a hyperbolic number $u + jv$, which transforms a hyperbolic number $F_{n+1} + jF_n$ into its neighboring hyperbolic number $F_{n+2} + jF_{n+1}$ from the sequence (3), the following equation (5) should be solved:

$$(F_{n+1} + jF_n)\,(u + jv) = (F_{n+2} + jF_{n+1}) \quad (5)$$

The solution to this equation (5) gives the following expressions (6) for components of the desired hyperbolic number $u + jv$:

$$u = F_{n+1}/F_n + (-1)^{n+1} {}^* F_{n-1} / (F_n{}^*(F_n^2 - F_{n-1}^2)),\ v = (-1)^n / (F_n^2 - F_{n-1}^2) \quad (6)$$

Now let us describe the results of the author's study of eigenvalues of the bisymmetric matrices in the parastichic hyperbolic sequence (4). Each of these matrices $[F_{n+1}, F_n; F_n, F_{n+1}]$ has two eigenvalues, which are equal to two Fibonacci numbers again: F_{n+2} and F_{n-1}. One can note that these eigenvalues are the sum and the difference of the Fibonacci components of the original hyperbolic number $F_{n+1} + jF_n$ since $F_{n+2} = F_{n+1} + F_n$ and $F_{n-1} = F_{n+1} - F_n$. The ratio F_{n+2}/F_{n-1} of such eigenvalues defines a new sequence (7) of Fibonacci ratios, which tend to φ^3 as n increases:

$$F_{n+2}/F_{n-1}: 3/1, 5/1, 8/2, 13/3, 21/5, 34/8, 55/13, \quad (7)$$

By analogy with expressions (1)–(4), such pair of eigenvalues F_{n+2} and F_{n-1} in (7) can be considered as components of a new hyperbolic number $F_{n+2} + jF_{n-1}$. In this case the sequence of ratios (7) is transformed into additive sequences (8), (9) reflecting linear notation of appropriate hyperbolic numbers and their matrix representations:

$$F_{n+2} + jF_{n-1}: 3 + j,\ 5 + j,\ 8 + j2,\ 13 + j3,\ 21 + j5,\ 34 + j8,\ 55 + j13, \quad (8)$$

$$\begin{vmatrix} F_{n+2}, F_{n-1} \\ F_{n-1}, F_{n+2} \end{vmatrix} : \begin{vmatrix} 3,1 \\ 1,3 \end{vmatrix}, \begin{vmatrix} 5,1 \\ 1,5 \end{vmatrix}, \begin{vmatrix} 8,2 \\ 2,8 \end{vmatrix}, \begin{vmatrix} 13,3 \\ 3,13 \end{vmatrix}, \begin{vmatrix} 21,5 \\ 5,21 \end{vmatrix} \quad (9)$$

Each of symmetric matrices $[F_{n+2}, F_{n-1}; F_{n-1}, F_{n+2}]$ of the sequence (9) has two eigenvalues $2F_{n+1}$ and $2F_n$, which are equal to two Fibonacci numbers multiplied by a factor 2. If such pair of eigenvectors are again represented in a form of appropriate hyperbolic number $2F_{n+1} + 2F_n j$, new sequences (10), (11) arise:

$$2F_{n+1} + j2F_n: 2(2 + j),\ 2(3 + j2),\ 2(5+3j),\ 2(8+5j),\ 2(13+8j),\ 2(21+13j), \quad (10)$$

$$2 * \begin{vmatrix} F_{n+1}, F_n \\ F_n, F_{n+1} \end{vmatrix}; 2^* \begin{vmatrix} 2,1 \\ 1,2 \end{vmatrix}; \begin{vmatrix} 2^* \end{vmatrix} \begin{vmatrix} 3,2 \\ 2,3 \end{vmatrix}; 2^* \begin{vmatrix} 5,3 \\ 3,5 \end{vmatrix}; 2^* \begin{vmatrix} 8,5 \\ 5,8 \end{vmatrix} \quad (11)$$

These sequences (10), (11) differ from the original sequences (3), (4) only by doubling all their members and they can be considered as a dichotomous reproduction of the original sequences (3), (4).

Applying to the new sequence (11) the described procedure of considering the eigenvalues of each of its bisymmetric matrices as parts of a new bisymmetric matrix, representing a new hyperbolic number, we arrive at doubling the sequence (11) or quadrupling the original parastichic sequence (3). It reveals an iterative algorithm for the dichotomous reproduction of the parastichic sequences (3), (4) again and again using a repeating the described procedure. This new algorithm is connected with formalisms of the theory of resonances and can be considered as related to the author's conception of the morphoresonance field (Petoukhov, 2016).

3. Bisymmetric matrices and Punnet squares of the Mendelian genetics

Heredity is the passing of traits from parent to offspring. Traits are controlled by genes. The different forms of a gene for a certain trait are called alleles. There are two alleles for every trait in diploid organisms. Alleles can be dominant or recessive. Each cell in an organism's body contains two alleles for every trait. One allele is inherited from the female parent and one allele is inherited from the male parent. For a prediction of inherited traits of offspring, a well-known method of Punnet squares is used, which is described in most textbooks of Mendelian genetics. It was introduced by the British geneticist R.C. Punnett in 1905 (Crew, 1968). Below a connection of Punnet squares with bisymmetric matrices representing hyperbolic numbers is analyzed.

Punnett squares represent Mendel's laws of inheritance of traits under poly-hybrid crosses. It is a simple method for predicting how alleles can be combined. The Punnett square shows combinations of dominant and recessive alleles of genes from parent reproductive cells – gametes, that is, it is a summary of every possible combination of one maternal allele with one paternal allele for each gene being studied in the cross. In a Punnett square, dominant and recessive alleles are usually represented by letters. An uppercase letter represents a dominant allele and a lowercase letter represents a recessive allele (for example, for dominant alleles symbols H, B, …and for recessive alleles symbols h, b, …). An organism is homozygous if it has identical alleles for a particular trait, for example, HH or hh. An organism is heterozygous if it has non-identical alleles for a particular trait, for example, Hh. In the classic way of constructing Punnett square, alleles of a maternal gamete are put on one side of the square (usually on the top) and alleles of a paternal gamete are put on the left side; the possible progeny are produced by filling the squares with one allele from the top and one allele from the left to produce the progeny genotypes. If only one trait is being considered in a genetic cross, the cross is called monohybrid. If two or three traits are being considered in a genetic cross, the cross is called dihybrid or trihybrid correspondingly. Fig. 1 shows examples of Punnet squares for monohybrid and dihybrid crosses of organisms under the laws of Mendel.

Let us show that Punnet squares are modeled by using tensor families of bisymmetric matrices, which represent hyperbolic numbers and have real eigenvalues. As known, the operation of the tensor (or Kronecker) product of any two square matrices V and W have the following property: the eigenvalues of matrix V⊗W are equal to a product of c_k*d_s, where c_k and d_s are eigenvalues of the matrices V and W (Bellman, 1960). This feature of the tensor inheritance of eigenvalues of the original matrices (or "parental" matrices) V and W can be conveniently represented in the form of "tables of inheritance".

For example, if a bisymmetric matrix V = [p, q; q, p] has eigenvalues H and h, then its second tensor power $V^{(2)}$ has 4 following eigenvalues: HH, Hh, hH, hh. If we consider the tensor product of two bisymmetric (4*4)-matrices $V^{(2)}$ and $W^{(2)}$, which have their eigenvalues HH, Hh, hH, hh and BB, Bb, bB, bb correspondingly, then the (16*16)-matrix $V^{(2)}\otimes W^{(2)}$ has 16 following eigenvalues: HHBB, HHBb, HhBB, HhBb, HHBb, HHbb, HhBb, Hhbb, HhBB, HhBb, hhBB, hhBb, HhBb, Hhbb, hhBb, hhbb. These sets of eigenvalues, which are produced by a tensor hybridization of two bisymmetric matrices, can be represented in a form of square tables (Fig. 2), which are similar to appropriate Punnet squares for monohybrid and dihybrid crosses of organisms under the laws of Mendel (Fig. 3). If the matrices V and W represent appropriate oscillatory systems then eigenvalues of matrices V, W, $V^{(2)}$, $W^{(2)}$, and $V^{(2)}\otimes W^{(2)}$ determine resonant frequencies (or spectra) of these oscillatory systems. Briefly speaking, this model approach connects Mendelian genetics with eigenvalues of bisymmetric matrices, representing hyperbolic numbers, and with the theory of resonances of oscillatory systems having many degrees of freedom. This approach can be applied analogically to other polyhybrid crosses.

This formal analogy - between Punnett squares of combinations of alleles and tables of tensor inheritance of real eigenvalues of bisymmetric matrices of vibrosystems - generates the following idea:

- alleles of genes and their combinations can be interpreted as eigenvalues of bisymmetric (2^n*2^n)-matrices from tensor families of matrices of oscillatory systems. For genetic systems, this model approach focuses attention on the possible importance of a particular class of mutually related resonances from tensor families of bisymmetric matrices, which represent hyperbolic numbers and play the role of biological "matrix archetypes".

4. The Weber-Fechner logarithmic law and hyperbolic rotations

The human body receives sensory information through various sensory organs: eyes, ears, nose, tactile receptors, taste buds, etc. These inherited organs are very different from each other in their appearance and biochemical composition. But the sensory information, entering the body through these organs, obeys the same law, called the main psychophysical law of Weber-Fechner. This logarithmic law declares the following: the intensity of the perception is proportional to the logarithm of stimulus intensity; it is expressed by the equation

$$p = k*\ln(x/x_0) = k*\{\ln(x) - \ln(x_0)\}, \qquad (12)$$

where p - the intensity of perception, x – stimulus intensity, x_0 - threshold stimulus, ln – natural logarithm, k – a weight factor. Because of this law, the power of sound in technology is measured on a logarithmic scale in decibels.

It is profitable for an organism, which is a single whole, to have the same typical algorithms at different levels of its functioning for optimal coordination of its parts. As known, mechanisms of resonances are very important for hearing and other sensory functions and they can serve as mechanisms of functional coordination of different parts in working systems. Below the author shows that the logarithmic law of Weber-Fechner (12) is structurally connected with hyperbolic rotations, which are represented by bisymmetric matrices. Such matrices are matrix representations of some hyperbolic numbers and are related formally to resonance frequencies of a particular class of oscillatory systems with two degrees of freedom.

Historically the natural logarithm was formerly termed the hyperbolic logarithm, as it corresponds to the area under a hyperbola (Klein, 2004). More precisely, the natural logarithm can be defined for any positive real number "a" as the area under the hyperbola $y = 1/x$ from 1 to a (Fig. 3, left). It means that two points of the hyperbola with their coordinates $(x, 1/x)$ and $(x_0, 1/x_0)$, where $x > 1$, $x_0 > 1$, define values of natural logarithms $\ln(x)$ and $\ln(x_0)$. Subtraction $\ln(x)$-$\ln(x_0)$ gives the intensity of the perception p in the expression (12) of the Weber-Fechner law (Fig. 3, right). A change of a stimulus intensity x_1 into a new stimulus intensity x_2 corresponds to a hyperbolic rotation, which shifts the hyperbola $y = 1/x$ along itself and transfer its point $(x_1, 1/x_1)$ to its point $(x_2, 1/x_2)$. Each of these two points defines its individual value of natural logarithm namely $\ln(x_1)$ and $\ln(x_2)$ and – in relation to the logarithmic law of Weber-Fechner (12) - corresponds to its individual value of the intensity of the perception $p_1 = k*\{\ln(x_1) - \ln(x_0)\}$ and $p_2 = k*\{\ln(x_2) - \ln(x_0)\}$. The change of the intensity of perception Δp in this case is given by the expression (13):

$$\Delta p = p_2 - p_1 = k*\{\ln(x_2) - \ln(x_0)\} - k*\{\ln(x_1) - \ln(x_0)\} = k*\ln(x_2/x_1) \qquad (13)$$

This explains a relation of hyperbolic rotations to modeling the logarithmic law of Weber-Fechner.

One can suppose that the innate logarithmic Weber–Fechner law is the law especially for the nervous system. But it is not so since its meaning is much wider because it holds true in many kinds of lower organisms without a nervous system in them: "*this law is applicable to chemo-tropical, helio-tropical and geo-tropical movements of bacteria, fungi and antherozoids of ferns, mosses, and phanerogams …. The Weber-Fechner law, therefore, is not the law of the nervous system and its centers, but the law of protoplasm in general and its ability to respond to stimuli*" (Shults, 1916, p.126). Correspondingly, hyperbolic rotations, connected with this logarithmic law and expressed by hyperbolic numbers, have fundamental importance for modeling a wide class of biological structures.

		Maternal gametes	
		H	h
paternal gametes	H	HH	Hh
	h	hH	hh

		maternal gametes			
		HB	Hb	hB	hb
pat. gam.	HB	HHBB	HHBb	HhBB	HhBb
	Hb	HHBb	HHbb	HhBb	Hhbb
	hB	HhBB	HhBb	hhBB	hhBb
	hb	HhBb	Hhbb	hhBb	hhbb

Fig. 1. Examples of Punnett squares for monohybrid and dihybrid crosses of organisms under the laws of Mendel. The abbreviation « pat. gam.» means « paternal « paternal gametes». Letters H and B denote dominant alleles; letters h and b denote recessive alleles.

		maternal spectrum	
		H	h
paternal spectrum	H	HH	Hh
	h	hH	hh

		maternal spectrum			
		HB	Hb	hB	hb
pat. sp.	HB	HHBB	HHBb	HhBB	HhBb
	Hb	HHBb	HHbb	HhBb	Hhbb
	hB	HhBB	HhBb	hhBB	hhBb
	hb	HhBb	Hhbb	hhBb	hhbb

Fig. 2. Tables of inheritance of eigenvalues of bisymmetric matrices in cases of tensor monohybrid and dihybrid hybridizations (see explanations in the text). The abbreviation « pat. sp.» means « paternal spectrum».

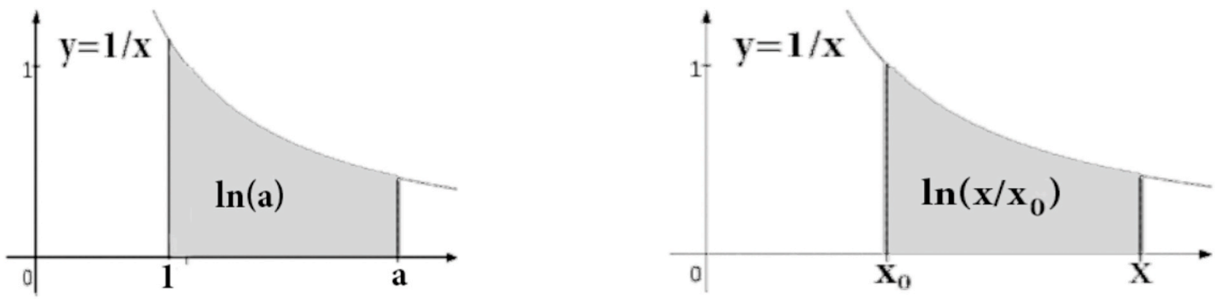

Fig. 3. Natural logarithm as a square under hyperbola $y = 1/x$. Left: ln(a) is equal to the square under the hyperbola from 1 to a. Right: ln($x/x0$) is equal to the square under the hyperbola from x_0 to x.

Hyperbolic rotations are particular cases of bisymmetric matrices representing of 2-dimensional hyperbolic numbers. The described analysis gives pieces of evidence that our sensory perception obeys the same structural principles as morphogenesis with its phyllotaxis laws and that these principles can be effectively modeling on the basis of hyperbolic numbers. One experimental fact can be added here about a structural analogy between morphogenesis and visual perception. Phyllotaxis laws are related to the golden ratio $\varphi = (1+5^{0.5})/2 = 1618$ …. The famous American neurophysiologist and one of the founders of cybernetics McCalloch experimentally shown that the golden ratio has an amazing aesthetical meaning in human visual perception (McCulloch, 1965, c. 395). He wrote that he spent two years measuring the person's ability to bring an adjustable oblong object to a preferred shape because he did not believe that human persons prefer the golden ratio or that they could recognize it. They prefer it and they can! In repeated experimental constructing the most pleasant forms, human persons come to the preference of the golden ratio and they can establish it. As McCulloch concluded one who is able to detect a difference in the twentieth of the length, area or volume, exposes this difference to 1:1, 618, and not to 1:1617 or 1:1619.

The idea about an inherited structural connection between morphogenesis, perception, and consciousness exist long ago. For example, the book (Nalimov, 2015 p. 115) claims: "*artificial intelligence could be brought closer to mathematical thinking if it were possible to realize the metrical properties of the human mind …. the consciousness itself is structured geometrically: any person in his existential aspects is geometric. … in our minds when constructing texts through which we perceive the World, something very similar to what happens in morphogenesis occurs. We are ready to see in the depths of consciousness the same geometric images that are revealed in morphogenesis*". The development of this line of thought is important, in particular, for the improvement of artificial intelligence systems and biotechnologies.

5. Hyperbolic numbers and hyperbolic geometry for modeling other biosystems

This Section notes some other biosystems whose inherited structures were mathematically modeled by different authors on the basis of the ideology of hyperbolic numbers and hyperbolic geometry.

In accordance with the pioneering work (Luneburg, 1950), the space of binocular visual perception is described by hyperbolic geometry. These findings were followed by many papers in various countries, where the idea of a non-Euclidean space of visual perception was extended and refined. The Luneburg approach was thoroughly tested in the work (Kienle, 1964). In the main series of his experiments, where about 200 observers were involved, Kienle obtained about 1300 visual patterns of various kinds. The experiments confirmed not only that the space of visual perception is described by hyperbolic geometry but also that the well-known Poincare disk (or conformal) model was an adequate model of that geometry. Kienle concluded his paper by writing: "*Poincare's model of hyperbolic space, applied for the first time for a mapping of the visual space, shows a reasonably good agreement with experimental results*" (Kienle, p. 399).

The works (Schelling, 1960, 1964) introduced a non-Euclidean metric to describe color perception and constructed a perceptual theory of relativity by analogy with the notion of a space-time manifold in the special theory of relativity.

Let us turn now to a question on an inherited ability of animals to locomotion. As known, newborn turtles and crocodiles, when they hatched from eggs, crawl with quite coordinated movements to water without any training from anybody. Celled organisms, which have no nervous systems and muscles, move by means of perfectly coordinated motions of cilia on their surfaces (the genetically inherited "dances of cilia"). In these inherited motions, a huge number of muscle fibers, nerve cells, contractile proteins, enzymes, and so forth are acting in concert, by analogy with the coordinated work of a plurality of parts of computers. The article (Smolyaninov, 2000) presents the results of 20 years of research by the author of the spatio-temporal features of locomotion in many animals and humans in different locomotor modes to substantiate his "locomotor theory of relativity" (Smolyaninov, 1984). These results give pieces of evidence that the spatio-temporal control of locomotion is related to the hyperbolic geometry of Minkowski.

The author shows the existence of a system of locomotor invariants and claims: «*The locomotor geochronometry is analogic to the well-known Minkowski geometry … The commonality of kinematic laws of walking in arthropods and humans, found in our comparative studies, confirms the commonality of kinematic tasks of the walking movements of all walkers — invertebrates and vertebrates —, which makes us think about the* *neurophysiological mechanisms of the phylogenetic invariance of kinematic control programs in view of the significant differences in leg constructions and the dynamics of their movements*" (Smolyaninov, 2000, p. 1094, 1018). In addition, this author put forward the concept of "relativistic brain" and relativistic biomechanics.

To conclude this Section, we note another analogy between inherited physiological systems and the considered type of multidimensional numbers. Bisymmetric matrix operators, representing 2^n-dimensional hyperbolic numbers and related to the theory of resonances, have a cruciform character since these matrices are symmetrical with respect to both diagonals. It is interesting that many genetic inherited constructions of physiological macrosystems including sensory-motion systems have a similar cruciform character. For example, the connection between the hemispheres of the human brain and the halves of human body possesses the similar cruciform character: the left hemisphere serves the right half of the body and the right hemisphere serves the left half of the body (Fig. 4) (Annett, 1992; Gazzaniga, 1995; Hellige, 1993). The system of optic cranial nerves from two eyes possesses the cruciform structures as well: the optic nerves transfer information about the right half of the field of vision into the left hemisphere of the brain, and information about the left half of field of vision into the right hemisphere. The same is held true for the hearing system (Penrose, 1989, Chapter 9). It is natural to think that these inherited physiological phenomena have relations with mentioned cruciform algebraic structures in the genetic code system.

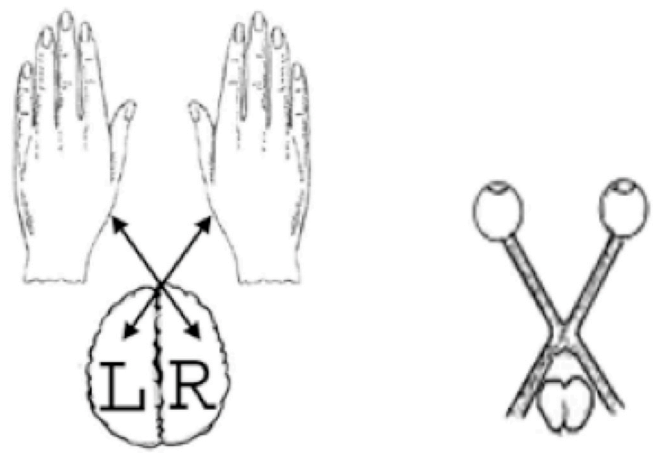

Fig. 4. The cruciform organization of some morpho-functional structures in a human body. On the left side: cruciform connections of brain hemispheres with the left and the right halves of a human body. On the right side: the cruciform structure of optic nerves from eyes in the brain.

6. Some concluding remarks

The value of mathematics in studies of natural systems can be compared with the value of glasses for a visually impaired person: when such a person looks at the world through good glasses, he begins to see many useful details and connections that he did not see before. No wonder Charles Darwin noted that people with an understanding of the great leading principles of mathematics seem to have an extra sensory organ in comparison with other people (Darwin, 1905).

This article presents materials about using the mathematics of multidimensional hyperbolic numbers for modeling of different inherited biosystems, which are parts of the holistic body. This modeling approach reveals hidden structural interconnections among these biosystems and leads deeper understanding of their commonality related to the commonality of their genetic basis. At the same time, it shows the structural connection of inherited biosystems with formalisms of the theory of resonances of oscillatory systems with many degrees of freedom. This complements works of many authors - from antiquity to the present day - on the importance of coordinated oscillations and resonances in the life of living organisms.

The harmonic progression, which is connected with the hyperbolic rules of cooperative organization of genomes (Petoukhov, 2020a,b), is a special recurrent numeric sequence. The principle of recurrent sequences in building many biological structures is known long ago. This principle should be studied further more deeply as one of the general principles of biological organization, including the extended approach called the "replication with variation", which is described in the book (Ji, 2018, p. 384–389) and which is applicable, in particular, to phyllotaxis laws having connections with Fibonacci numbers.

It is also interesting to note the following. This modeling approach on the basis of bisymmetric matrices, representing hyperbolic numbers, reveals that the structural commonality of different genomes is related to the harmonic progression $1/n$ and the harmonic mean (Petoukhov, 2020a,b), which are long known in arts and culture. They are used in the Pythagorean teaching on the musical harmony and the aesthetics of proportions. For example, in the theory of musical harmony, the tuning of musical intervals as whole number ratios of frequencies is known as the pure intonation tuning (or simply, the pure tuning). Any interval tuned in this way is called a just interval. Just intervals (and chords created by combining them) consist of members of a single harmonic series. The harmonic progression exists also in ratios of overtones wavelengths to a wavelength of a basic tone. But the harmonic progression is important not only in music. At least from the time of the Pythagorean doctrine of the aesthetics of proportions, an idea exists that the aesthetics of one art is simultaneously aesthetics of another art; only the material is different. In light of this, architecture has long been interpreted as frozen music and music as dynamic architecture. Speculation about the possible genetic basis of a number of aesthetic parallels in various arts arises. The deep connection of inherited biosystems with the algebraic harmonic progression allows saying about the algebra-harmonic organization of living bodies.

The hyperbolic rules of genomes were obtained through the consideration of each long nucleotide DNA test as a multilevel text containing many interconnected texts, each of which consists of oligomers ("words") of fixed length specifical for each level (Petoukhov, 2020a,b). It seems that a creation of a general theory of noise-immune transferring of information by similar multilevel texts is interesting and perspective.

In addition, the using - for biosystems modeling - of multidimensional hypercomplex numbers, which are one of the main tools in modern mathematical natural sciences having a great collection of ideas and achievements, opens a bridge between these sciences and biology for their mutual enrichment.

Declaration of competing interest

None.

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