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SYMMETRIES IN MOLECULAR-GENETIC SYSTEMS AND MUSICAL HARMONY

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Abstract: *The Moscow State Conservatory by P.I. Tchaikovsky has recently created a special "Center for interdisciplinary researches of musical creativity". One of the main tasks of this center is to study genetic musical scales from different viewpoints including new opportunities for composers and for musical therapy. This article is devoted to scientific aspects of the genetic musical scales, which are based on symmetric features of molecular ensembles of genetic systems. These musical scales were revealed in a course of symmetrologic study of representations of molecular-genetic ensembles in a united form of mathematical matrices (Kronecker families of genetic matrices). This study has discovered a relation of genetic systems with the golden section and Fibonacci numbers, which play role in a hierarchical system of these musical scales and which are well known in biological phyllotaxis laws and in aesthetics of proportions. Some historical and biological aspects of musical harmony are also considered.*

Keywords: symmetry, musical harmony, genetic code, golden section, Fibonacci numbers.

1. ABOUT THE GENETIC CODING SYSTEM AND GENETICALLY INHERITED PERCEPTION OF MUSIC

From ancient times, understanding the phenomenon of music and building musical structures were associated with mathematics. The creator of the first computer G.Leibniz wrote: "Music is a secret arithmetical exercise and the person who indulges in it does not realize that he is manipulating numbers" and "music is the pleasure the human mind experiences from counting without being aware that it is counting" (http://thinkexist.com/quotes/g._wilhelm_leibniz/).

The range of human sound perception contains an infinite set of sound frequencies. Pythagoras has discovered that certain mathematical rules, based on integers, allow separating - from this infinite set of frequencies - a discrete set of frequencies, which determine the harmonious sound set. In other words, certain combinations of sounds from this set are perceived by living organisms as pleasant for hearing (consonances). In addition, Pythagoras has linked the phenomenon of the harmonic sounds with the parameters of a physical object: oscillation frequencies of stretched string, the length of which is varied in accordance with appropriate numerical rules. But these discoveries by Pythagoras say nothing about the fact that other discrete sets of sound frequencies may exist, which will also form harmonious sets of sounds.

This article describes some results of researches of molecular ensembles of the genetic coding system. The results reveal that sets of parameters of this molecular genetic system are related with the well-known Pythagorean musical scale and also with a hierarchy of special mathematical sets. This hierarchy can be interpreted and used as the base of a new system of musical scales, because appropriate sets of sound frequencies may possess harmonic properties for human hearing. According to our assumption, it seems to be essential that these musical systems be connected with the molecular-genetic system because the phenomenon of musical perception is inherited.

The scientific studies of physiological mechanisms of musical perception took place long ago. One can find the review on this topic in the article (Weinberger, 2004). Beginning with 4-months old infants turn to a source of pleasant sounds (consonances) and turn aside a source of unpleasant sounds (dissonances). The human brain does not possess a special center of music. The feeling of love to music seems to be dispersed in the whole organism. The musical sound addresses to all in the person, or to person's archetypes. There are known data that the first shout of the baby, who has been born, corresponds to sounds on frequency of the music note "la" (440 Hz) irrespective of its timbre and of loudness, as a rule. (http://www.rods.ru/Html/Russian/MoreResonance.html). This frequency is used traditionally for tuning musical instruments by means of a tuning fork.

This speaks about certain biological unification of musical sounds. According to statistics, physical reactions to music (in the form of skin reactions, tears, laugh, etc.) arise in 80 % of adult people. Animals also are not indifferent to human music. All such data show that the perception of music has biological essence and that the feeling of musical harmony is based on inborn mechanisms. Therefore it is necessary to search for connections of the genetic system with musical harmony. This article presents such a search.

It can be mentioned that thoughts about the key significance of musical harmony in the organization of the world exist from ancient time. For example, Pythagoreans thought about musical intervals in the planetary system and in all around. J. Kepler wrote the famous book *Harmonices Mundi*, etc. Modern atomic physics found the harmonic ratios in spectral series by T. Lyman in the atom of hydrogen, which has been named "music of atomic spheres" by A. Einstein and A. Sommerfeld (Voloshinov, 2000). The importance of Pythagorean ideas about a role of musical harmony was emphasized also by the Nobel prize winner in physics R. Feynman (1963, v. 4, Chapter 50).

The living substance is compared with crystals frequently. For example, E. Schrödinger (1955) named it "aperiodic crystal". Whether annals of modern science contain any data about a connection of musical harmony with crystals? Yes, such data exist (see, for example, the book (Berger, 1997, p. 270-281).

In 1818, C.S. Weiss, who discovered crystallographic systems and who was one of founders of crystallography, emphasized a musical analogy in crystallographic systems. He investigated ratios among segments, which are formed by faces of crystals of the cubic system. Weiss has shown that these ratios are identical absolutely to ratios between musical tones.

In 1829, J. Grassman, who wrote a well-known book "Zur Physischen Kristallonomie und Geometrishen Combinationslehre" and developed many mathematic methods in crystallography, noted impressive musical analogies in the field of crystallography. The statement is about many analogies described by him between ratios of musical tones and segments, formed by faces of the same zone of crystals. According to his figurative expression, "crystal polyhedron is a fallen asleep chord - a chord of the molecular fluctuations made in time of its formation" (from (Berger, 1997, p. 270)).

At the end of 1890's the outstanding crystallographer V. Goldschmidt returned to the same ideas. The prominent Russian mineralogist and geochemist A.E. Fersman wrote about his thematic publications: "These works represent the historical page in crystallography, which has lead Goldschmidt to revealing by him laws of harmonic ratios. Goldschmidt has extended these laws logically from the world of crystals into the world of other correlations in the regions of paints, colors, sounds and even biological correlations. It has become one of the most favourite themes of philosophical researches by Goldschmidt" (from (Berger, 1997, p. 270)). This list of such historical examples can be continued.

Taking into account, that Shrödinger named a living substance as aperiodic crystal and that the classicists of crystallography emphasized a connection between crystal structures and musical harmony, it seems natural to try to find traces of musical harmony in living substance as well. This idea about a possible participation of musical harmony in the organization of biological organisms is not new for modern biophysics. For example, the famous Russian biophysicist, S. Shnoll (1989) wrote: "*From possible consequences of interaction of macromolecules of enzymes, which are carrying out conformational (cyclic) fluctuations, we shall consider pulsations of pressure - sound waves. The range of numbers of turns of the majority of enzymes corresponds to acoustic sound frequencies. We shall consider ... a fantastic picture of "musical interactions" among biochemical systems, cells, bodies, and a possible physiological role of these interactions. …… It leads to pleasant thoughts about nature of hearing, about an origin of musical perception and about many other things, which already belong to the area of biochemical aesthetics*". This term "biochemical aesthetics", proposed by Schnoll, reflects materials of our article.

Let us recall some fundamental notions of the theory of musical harmony. Each musical note is characterized by its certain frequency of sounding. For musical melody, a ratio between frequencies of neighboring notes is important, but not the absolute values of frequencies of separate notes. For this reason the melody is easily distinguished irrespective of what acoustic range of frequencies it is produced in, for example, by child, woman or adult man with quite different voices. An aggregate of frequency values between sounds in musical system is named a musical scale. The same note, for example, the note "do" is distinguished by the person as the same if its frequency is increased or reduced twice i.e., if it belongs to another octave. The interval of frequencies from some note frequency f_0 up to frequency $2*f_0$ is named an octave. Each note "do" is considered usually as the beginning of the appropriate octave. For example, the first octave reaches from frequency 260 Hz approximately (the note "do" of the first octave) up to the double frequency 520 Hz (the note "do" of the second octave).

Small quantity of discrete frequencies of the octave diapason is traditionally used for musical notes only. The notes, which correspond to these frequencies, form a certain sequence in ascending order of frequencies. A musical scale represents a sequence of numerical values ("interval values") between frequencies of the adjacent notes (musical tones).

For Europeans the idea of musical harmony of a universe is connected basically with the name Pythagoras and his school. After ancient thinkers (first of all, ancient Chinese thinkers) Pythagoreans considered that the world is arranged by principles of musical harmony. The Pythagorean musical scale, which is based on the quint ratio 3:2, played the main role in these views. One should note that this musical scale was known in Ancient China long before Pythagoras, who has presumably got acquainted with it in his life in Egypt and Babylon (the analysis of these questions is presented in detail in the book (Needham, v.4, 1962)). In Ancient China this quint music scale had a cosmic meaning connected with "The Book of Changes" ("I Ching"): numbers 2 and 3 were named "numbers of Earth and Heaven" there. After Ancient China, Pythagoreans considered numbers 2 and 3 as the female and male numbers, which can give birth to new musical tones in their interconnection. According to some data, the quint system of the musical scale is the most ancient among known systems in the history of musical scales (http://www.arbuz.uz/t_octava.html).

Ancient Greeks attached an extraordinary significance to the search of the quint 3:2 in natural systems because of their thoughts about musical harmony in the organization of the world. For example, the great mathematician and mechanician Archimedes considered the detection of the quint 3:2 between volumes and areas of a cylinder and a sphere entered in it (Voloshinov, 2000) as the best result of his life. Just these geometrical figures with the quint ratio were pictured on his gravestone according to Archimedes testament. And due to these figures Cicero has found Archimedes' grave later, 200 years after his death. This article demonstrates, in particular, the connection of the Kronecker family of the genomatrices of hydrogen bonds with the Pythagorean musical scale based on the quint ratio 3:2.

2. NUMERIC GENOMATRICES OF HYDROGEN BONDS

One of the effective methods of cognition of a complex natural system, including the genetic coding system, is the investigation of symmetries.

Modern science knows that deep knowledge about phenomenological relations of symmetry among separate parts of a complex natural system can tell many important things about the evolution and mechanisms of these systems. This article studies some symmetry properties of the genetic coding system by means of matrix representation and analysis of molecular ensembles of the genetic system. An initial choice of such a form of presentation of molecular ensembles of the genetic code is explained by the following main reasons.

Information is usually stored in computers in the form of matrices.

- The genetic coding system provides noise-immunity properties; noise-immunity codes are constructed on the basis of matrices.

- Genetic molecules obey principles of quantum mechanics, which utilizes matrix operators. A connection between genetic matrices and these matrix operators can be revealed. The significance of matrix approach is emphasized by the fact that quantum mechanics has arisen in a form of matrix mechanics by W. Heisenberg.

- Complex and hypercomplex numbers, which are utilized in physics and mathematics, possess matrix forms of their presentation. The notion of number is the main notion of mathematics and mathematical natural sciences. In view of this, investigation of a possible connection of the genetic code to multi-dimensional numbers in their matrix presentations can lead to very significant results.

- Matrix analysis is one of the main investigation tools in mathematical natural sciences. The study of possible analogies between matrices, which are specific for the genetic code, and famous matrices from other branches of sciences can be heuristic and useful.

- Matrices, which are a kind of union of many components in a single whole, are subordinated to certain mathematical operations, which determine substantial connections between collectives of many components. Such connections can be essential for collectives of genetic elements of different levels as well.

In history of science, the first publication about matrix representation of molecular ensembles of the genetic coding system was the work

(Konopel'chenko, Rumer, 1975), which studied symmetries in the genetic system. It represented the genetic alphabet *A* (adenine), *C* (cytosine), *G* (guanine), *T* (thymine) and the set of 16 duplets in a form of the two square matrices [C G; T A] and $[C\ G;\ T\ A]^{(2)}$ respectively (here 2 in brackets means the Kronecker power).

In our article we continue studying the molecular genetic system in its matrix forms of representation, which has given some interesting results in the last 10 years (Petoukhov, 2005, 2008; Petoukhov, He, 2010). In these works we studied the Kronecker family of genetic matrices $[C\ T;\ A\ G]^{(n)}$ (here "*n*" in brackets means the Kronecker power), the first representatives of which are shown in Fig. 1.

[C T; A G]=

C	T
A	G

; $[C\ T;\ A\ G]^{(2)}$ =

CC	CT	TC	TT
CA	CG	TA	TG
AC	AT	GC	GT
AA	AG	GA	GG

$[C\ T;\ A\ G]^{(3)}$ =

CCC	CCT	CTC	CTT	TCC	TCT	TTC	TTT
CCA	CCG	CTA	CTG	TCA	TCG	TTA	TTG
CAC	CAT	CGC	CGT	TAC	TAT	TGC	TGT
CAA	CAG	CGA	CGG	TAA	TAG	TGA	TGG
ACC	ACT	ATC	ATT	GCC	GCT	GTC	GTT
ACA	ACG	ATA	ATG	GCA	GCG	GTA	GTG
AAC	AAT	AGC	AGT	GAC	GAT	GGC	GGT
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

Figure 1: The first members of the Kronecker family of genetic symbolic matrices $[C\ T;\ A\ G]^{(n)}$. Here A, C, G and T are adenine, cytosine, guanine and thymine correspondingly.

Numeric genomatrices can be derived from the replacement of each symbol A, C, G, T of the nitrogenous bases in the symbolic genomatrices $[C\ T;\ A\ G]^{(n)}$ (Figure 1) by quantitative parameters of these bases. For example, let us consider the genomatrices of hydrogen bonds of these nitrogenous bases. The hydrogen bonds 2 and 3 of complementary letters of the genetic alphabet are suspected for their important information meaning by different authors for a long time. In addition, hydrogen plays the main role in the composition of our Universe, where 93% hydrogen atoms exist among all kinds of atoms and where “chemical influence of omnipresent hydrogen is the defining factor” (Ponnamperuma, 1972). Thus the investigation of a possible meaning of hydrogen bonds in genetic information deserves special interest.

The complementary letters C and G have 3 hydrogen bonds (C = G = 3) and the complementary letters A and T have 2 hydrogen bonds (A = T= 2). Let us replace each multiplet in the Kronecker family of the genomatrices [C T; A G]$^{(n)}$ by the product of these numbers of its hydrogen bonds. In this case, we get the Kronecker family of numeric matrices [3 2; 2 3]$^{(n)}$. For example, the triplet CAT will be replaced by number 12 (=3*2*2) in the genomatrix [3 2; 2 3]$^{(3)}$. Figure 2 demonstrates the three initial genomatrices from this Kronecker family of genomatrices [3 2; 2 3]$^{(n)}$ constructed in this way. Numeric characteristics of each genomatrix [3 2; 2 3]$^{(n)}$ are connected with the quint ratio 3:2; for this reason we name such genomatrices as quint genomatrices conditionally.

$$Q = \begin{vmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} \end{vmatrix}; \quad Q^{(2)} = \begin{vmatrix} \mathbf{9} & \mathbf{6} & \mathbf{6} & \mathbf{4} \\ \mathbf{6} & \mathbf{9} & \mathbf{4} & \mathbf{6} \\ \mathbf{6} & \mathbf{4} & \mathbf{9} & \mathbf{6} \\ \mathbf{4} & \mathbf{6} & \mathbf{6} & \mathbf{9} \end{vmatrix}; \quad Q^{(3)} = \begin{vmatrix} \mathbf{27} & \mathbf{18} & \mathbf{18} & \mathbf{12} & \mathbf{18} & \mathbf{12} & \mathbf{12} & \mathbf{8} \\ \mathbf{18} & \mathbf{27} & \mathbf{12} & \mathbf{18} & \mathbf{12} & \mathbf{18} & \mathbf{8} & \mathbf{12} \\ \mathbf{18} & \mathbf{12} & \mathbf{27} & \mathbf{18} & \mathbf{12} & \mathbf{8} & \mathbf{18} & \mathbf{12} \\ \mathbf{12} & \mathbf{18} & \mathbf{18} & \mathbf{27} & \mathbf{8} & \mathbf{12} & \mathbf{12} & \mathbf{18} \\ \mathbf{18} & \mathbf{12} & \mathbf{12} & \mathbf{8} & \mathbf{27} & \mathbf{18} & \mathbf{18} & \mathbf{12} \\ \mathbf{12} & \mathbf{18} & \mathbf{8} & \mathbf{12} & \mathbf{18} & \mathbf{27} & \mathbf{12} & \mathbf{18} \\ \mathbf{12} & \mathbf{8} & \mathbf{18} & \mathbf{12} & \mathbf{18} & \mathbf{12} & \mathbf{27} & \mathbf{18} \\ \mathbf{8} & \mathbf{12} & \mathbf{12} & \mathbf{18} & \mathbf{12} & \mathbf{18} & \mathbf{18} & \mathbf{27} \end{vmatrix}$$

Figure 2 The beginning of the family of the quint genomatrices [3 2; 2 3]$^{(n)}$, which are based on the product of numbers of hydrogen bonds (C=G=3, A=T=2)

3 THE NUMERIC GENOMATRICES AND THE GOLDEN SECTION

In biology, a genetic system provides the self-reproduction of biological organisms in their generations. In mathematics, the “golden section” (or the “divine proportion”) and its properties were a mathematical symbol of self-reproduction from the Renaissance, and they were studied by Leonardo da Vinci, J. Kepler and many other prominent thinkers (see details in (Darvas, 2007; Shubnikov, Koptsik, 2005) and in the website “Museum of Harmony and Golden Section” by A. Stakhov, www.goldenmuseum.com).

Is there any connection between these two systems? Yes, and this article demonstrates such unexpected connection. The golden section is the value $\varphi = (1+5^{0.5})/2 = 1.618\ldots$ (Sometimes the inverse of this value is called the golden section in literature). If the simplest quint genomatrix [3 2; 2 3] is raised to the power 1/2 in the ordinary sense (that is, if we take the square root), the result is the bi-symmetric matrix $[\varphi \;\; \varphi^{-1}; \varphi^{-1} \;\; \varphi] = [3\ 2;\ 2\ 3]^{1/2}$, the matrix elements of which are equal to the golden section and to its inverse value. And if any other quint genomatrix $[3\ \ 2;\ 2\ \ 3]^{(n)}$ is raised to the power ½ in the ordinary sense, the result is the bi-symmetric matrix $[\varphi \;\; \varphi^{-1}; \varphi^{-1} \;\; \varphi]^{(n)} = ([3\ 2;\ 2\ 3]^{(n)})^{1/2}$, the matrix elements of which are equal to the golden section in various integer powers with elements of symmetry among these powers (Figure 3).

Here one can remind what does it mean: square root of a nonsingular square matrix M? It means such square matrix $M^{1/2}$, the second power of which is equal to the initial matrix M: $(M^{1/2})^2 = M$. Many known kinds of software (for example, MathLab) allow receiving square roots from nonsingular square matrices. Let us demonstrate here that for example the golden genomatrix $[\varphi \;\; \varphi^{-1}; \varphi^{-1} \;\; \varphi]$ is the square root from the quint genomatrix [3 2; 2 3]. Really, using ordinary rules of matrix multiplication, we get: $[\varphi,\ \varphi^{-1}; \varphi^{-1},\ \varphi]*[\varphi,\ \varphi^{-1}; \varphi^{-1},\ \varphi] = [\varphi*\varphi+\varphi^{-1}*\varphi^{-1},\ \ \varphi*\varphi^{-1} + \varphi^{-1}*\varphi; \varphi^{-1}*\varphi+\varphi*\varphi^{-1},\ \ \varphi^{-1}*\varphi^{-1} +\varphi*\varphi] = [\ 3,\ \ 2;\ 2,\ \ 3]$. Similar results can be checked for other corresponding pairs of the genomatrices: $([\varphi \;\; \varphi^{-1}; \varphi^{-1} \;\; \varphi]^{(n)})^2 = [3\ 2;\ 2\ 3]^{(n)}$. Matrices with matrix elements, all of which are equal to the golden section φ in different integer powers only, can be referred to as "golden matrices". Figuratively speaking, the quint genomatrices $[3\ 2;\ 2\ 3]^{(n)}$ have the secret substrate from the golden matrices $[\varphi \;\; \varphi^{-1}; \varphi^{-1} \;\; \varphi]^{(n)}$ (below we will explain a deep geometrical relationship between the quint matrices and the golden matrices, which represent square roots from them).

$$[3\ 2;\ 2\ 3]^{1/2} = \begin{vmatrix} \varphi & \varphi^{-1} \\ \varphi^{-1} & \varphi \end{vmatrix} \; ; \qquad ([3\ 2;\ 2\ 3]^{(2)})^{1/2} = \begin{vmatrix} \varphi^{2} & \varphi^{0} & \varphi^{0} & \varphi^{-2} \\ \varphi^{0} & \varphi^{2} & \varphi^{-2} & \varphi^{0} \\ \varphi^{0} & \varphi^{-2} & \varphi^{2} & \varphi^{0} \\ \varphi^{-2} & \varphi^{0} & \varphi^{0} & \varphi^{2} \end{vmatrix}$$

$([3\ 2;\ 2\ 3]^{(3)})^{1/2} =$

φ^{3}	φ^{1}	φ^{1}	φ^{-1}	φ^{1}	φ^{-1}	φ^{-1}	φ^{-3}
φ^{1}	φ^{3}	φ^{-1}	φ^{1}	φ^{-1}	φ^{1}	φ^{-3}	φ^{-1}
φ^{1}	φ^{-1}	φ^{3}	φ^{1}	φ^{-1}	φ^{-3}	φ^{1}	φ^{-1}
φ^{-1}	φ^{1}	φ^{1}	φ^{3}	φ^{-3}	φ^{-1}	φ^{-1}	φ^{1}
φ^{1}	φ^{-1}	φ^{-1}	φ^{-3}	φ^{3}	φ^{1}	φ^{1}	φ^{-1}
φ^{-1}	φ^{1}	φ^{-3}	φ^{-1}	φ^{1}	φ^{3}	φ^{-1}	φ^{1}
φ^{-1}	φ^{-3}	φ^{1}	φ^{-1}	φ^{1}	φ^{-1}	φ^{3}	φ^{1}
φ^{-3}	φ^{-1}	φ^{-1}	φ^{1}	φ^{-1}	φ^{1}	φ^{1}	φ^{3}

Figure 3 The beginning of the Kronecker family of the golden matrices $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(n)}$ = $([3\ 2;\ 2\ 3]^{(n)})1/2$, where $\varphi = (1+5^{0.5})/2$ = 1, 618… is the golden section

The mentioned matrix elements of the matrix $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(n)} = ([3\ 2;\ 2\ 3]^{(n)})^{1/2}$ can be constructed from a combination of φ and φ^{-1} directly by the following algorithm. We take a corresponding multiplet of the genomatrix $[C\ \ T;\ A\ \ G]^{(n)}$ and change its letters C and G to φ. Then we take letters A and T in this multiplet and change each of them to φ^{-1}. As a result, we obtain a chain with "n" links, where each link is φ or φ^{-1}. The product of all such links gives the value of corresponding matrix elements in the matrix $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(n)}$. For example, in the case of the matrix $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(n)}$, let us calculate a matrix element, which is disposed at the same place as the triplet CAT in the matrix $[C\ \ T;\ A\ \ G]^{(3)}$. According to the described algorithm, one should change the letter C to φ and the letters A and T to φ^{-1}. In the considered example, we obtain the following product: $(\varphi * \varphi^{-1} * \varphi^{-1}) = \varphi^{-1}$. This is the desired value of the considered matrix element for the matrix $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(3)}$ on Figure 3.

A ratio between adjacent numbers in numerical sequences inside each of such matrices $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(n)}$ (for example, $\ldots\varphi^{3}, \varphi^{1}, \varphi^{-1}, \varphi^{-3}\ \ldots$) is equal to φ^2 (or φ^{-2}) always. The same ratio φ^2 exists in regular 5-stars (Figure 4) as a ratio between sides of the adjacent stars entered in each other. Below we will use the name "pentagram musical scales" for new musical scales connected with the golden genomatrices. In view of this, let us remind that the pentagram and its metaphysical associations were explored by the Pythagoreans who considered it an emblem of perfection and health. Pythagoreans swore by it and used the pentagram as a distinctive sign of belonging to their community. But the pentagram has been known long before Pythagoras since ancient times as a sign that protects from all evil, so in Ancient Babylon it depicted on the doors of stores and warehouses to protect goods from damage and theft. It was also a sign of power and was used on the royal seals. The first known images of pentagrams date back to around 3500 BC, they were found at the territory of Ancient Mesopotamia. For early Christians, the pentagram was a reminder about the five wounds of Christ, from the crown of thorns on his forehead, and from the nails in the hands and feet.

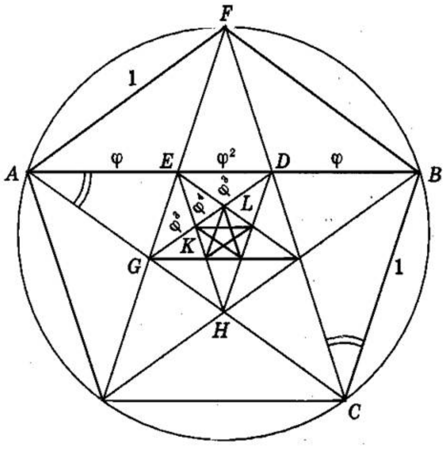

Figure 4 Sizes of pentagrams, which are entered in each other, differ by scale factor $\boldsymbol{\varphi^2}$ (or $\boldsymbol{\varphi^{-2}}$)

Let us remind that the value φ^2 (or φ^{-2}) is also well known in another genetically inherited phenomena of biological organisms, which are united under the term "phyllotaxis laws" (authors don't know how these two cases of realization of φ^2 are interconnected by biological mechanisms). Hundreds of books and articles around the world are devoted to these genetically inherited laws, which are connected with Fibonacci numbers and the golden section and which describe genetically inherited configurations of a huge number of living bodies at different levels and branches of biological evolution (see the review in the book (by Jean, 2010)). For example, leave arrangements in the cases of a lime tree, an elm tree, a beech are characterized by the ratio 2/1; in cases of an alder tree, a nut-tree, a vine, a sedge – by the ratio 3/1; in the cases of a raspberry, a pear tree, a poplar, a barberry – by the ratio 8/3; in the cases of an almond tree, a sea-buckthorn – by the ratio 13/5; cones of coniferous trees correspond to ratios 21/8, 34/13, 55/21 in various cases. All of these integer numbers are Fibonacci numbers, and the sequence of these ratios tends to the value φ^2 = 2,618… . The ideal angle in phyllotaxis laws, which is termed as "the Fibonacci angle", is equal to φ^{-2} (Jean, 2010, section 2.2.1).

The golden section is presented in 5fold-symmetrical objects of biological bodies (flowers, etc.), which are presented widely in the living nature but which are forbidden in classical crystallography. It exists as well in many figures of modern generalized crystallography: quasi-crystals by D. Shechtman, R. Penrose's mosaics, dodecahedra of ensembles of water molecules, icosahedral figures of viruses, biological phyllotaxis laws, etc. (Darvas, 2007). The article (Carrasco et al, 2009) shows that about 1-nm-wide ice chains that nucleate on metal surfaces Cu(110) are built from a face-sharing arrangement of water pentagons. The pentagon structure is favored over others because it maximizes the water–metal bonding while maintaining a strong hydrogen-bonding network. It reveals an unanticipated structural adaptability of water–ice films.

In recent years, unexpected connections are discovered between the golden section and micro-world of quantum mechanics, which includes genetic molecules. The article (Coldea et al., 2010) describes that the chain of atoms in certain circumstances acts like a nanoscale guitar string. The journal “Science Daily” gives a special title in its information about this discovery: “Golden Ratio Discovered in Quantum World: Hidden Symmetry Observed for the First Time in Solid State Matter”. The principal author of this paper R.Coldea speaks: “*Here the tension comes from the interaction between spins causing them to magnetically resonate. For these interactions we found a series (scale) of resonant notes: the first two notes show a perfect relationship with each other. Their frequencies (pitch) are in the ratio of 1.618…, which is the golden ratio famous from art and architecture. ... It reflects a beautiful property of the quantum system - a hidden symmetry. Actually quite a special one called E8 by mathematicians, and this is its first observation in a material*" (http://www.sciencedaily.com/releases/2010/01/100107143909.htm).

The new theme of the golden section in genetic matrices seems to be important because many physiological systems and processes are connected with it. It is known that proportions of a golden section characterize many physiological processes: cardio-vascular processes, respiratory processes, electric activities of brain, locomotion activity, etc. The golden section is described and is investigated for a long time in phenomena of aesthetic perception as well. Taking into account these facts, the golden section should be considered as the candidate for the role of one of base elements in an inherited interlinking of the physiological subsystems, which provides unity of an organism. The matrix relation between the golden section φ and significant parameters of genetic codes testifies in a favor of a molecular-genetic clue providing such physiological phenomena. One can hope that the algebra of bi-symmetric genetic matrices, which are connected with the theme of the golden section, will be useful for explanation and the numeric forecast of separate parameters in different physiological sub-systems of biological organisms with their cooperative essence and golden section phenomena.

One should emphasize the deep geometrical sense of the connection between the quint genomatrices $[3\ 2;\ 2\ 3]^{(n)}$ and golden genomatrices $[\varphi\ \ \varphi^{-1};\ \varphi^{-1}\ \varphi]^{(n)}$. This connection deals with the notion of "metric tensor", which is the main notion of Riemannian geometry (all other notions of Riemannian geometry - curvature tensor, geodesic lines, etc. – can be deduced from this main notion) (Rashevsky, 1964; http://en.wikipedia.org/wiki/Metric_tensor). The statement is that quint genomatrices [3 2; 2 3]$^{(n)}$ are metric tensors, and golden genomatrices [φ φ^{-1}; φ^{-1} φ]$^{(n)}$ are matrices of basic vectors of the frame of reference, on which this tensor is built. Let us explain it in more details. By definition, a metric tensor in n-dimensional affine space, where the operation of scalar product exists, is determined by means of a nonsingular matrix $||g_{ij}||$ with the condition of symmetry $g_{ij} = g_{ji}$ (Rashevsky, 1964, p. 157). Coordinates of the metric tensor g_{ij} are equal to the scalar products of pairs of the basic vectors e_i, e_j of the frame of reference, on which this tensor is built. The square root of the metric tensor $||g_{ij}||$ gives a square matrix, columns of which are basic vectors e_i of the frame of reference. But the quint matrices [3 2; 2 3]$^{(n)}$ satisfy the definition of metric tensors. Above we took the square root from this quint metric tensor [3 2; 2 3]$^{(n)}$, and as a result we received golden genomatrices [φ φ^{-1}; φ^{-1} φ]$^{(n)}$. It means that the metric tensors [3 2; 2 3]$^{(n)}$ are built on the corresponding bunches of the "golden" vectors (as their basic vectors of the frames of reference), all components of which are equal to the golden section φ in integer power. For example, the genomatrix [3 2; 2 3] can be interpreted as a metric tensor, which is built on a special affine frame of reference. This frame consists of two basic vectors: the golden vector e_1 with coordinates (φ, φ^{-1}) and the golden vector e_2 with coordinates (φ^{-1}, φ). These two golden vectors coincide with the columns in the golden genomatrix [φ φ^{-1}; φ^{-1} φ]. Scalar products of pairs of these vectors are equal to the components of the quint genomatrix [3 2; 2 3]: $<e_1, e_1> = \varphi*\varphi + \varphi^{-1}*\varphi^{-1} = 3$; $<e_1, e_2> = \varphi*\varphi^{-1} + \varphi^{-1}*\varphi = 2$; $<e_2, e_1> = \varphi^{-1}*\varphi + \varphi*\varphi^{-1} = 2$, $<e_2, e_2> = \varphi^{-1}*\varphi^{-1} + \varphi*\varphi = 3$.

One additional remark is the following. To interpret correctly the matrix [3 2; 2 3] as a metric tensor of a 2-dimensional plane, one should show a group of transformations, in relation to which this matrix plays a role of a tensor. In the considered case, for example, group of rotations can play can be taken for a tensor because their transformations conserve values of scalar products of the frame golden vectors (though the coordinates of these vectors are changed under such transformations). A similar situation holds true for other corresponding pairs of the quint genomatrices [3 2; 2 3]$^{(n)}$ and golden genomatrices [φ φ^{-1}; φ^{-1} φ]$^{(n)}$. In result, one can say that the considered Kronecker families of the quint genomatrices [3 2; 2 3]$^{(n)}$ and golden genomatrices [φ φ^{-1}; φ^{-1} φ]$^{(n)}$ are closely connected from a geometrical point of view or, in other words, they form a geometric organic whole. It should be added that the Riemannian geometry is very essential to study genetically inherited curved surfaces and lines of biological bodies: these curvilinear configuration endowed internal metric that is described by means of the Riemannian geometry (in view of this, some mathematical models of biological morphogenesis can be developed on the base of this geometry and its metric tensors).

The molecular system of the genetic alphabet is constructed by nature in such manner that not only numeric parameters of hydrogen bonds lead to the quint and golden genomatrices but some other significant parameters of genetic molecules lead also to quint and golden matrices by analogy. For example, the quantities of atoms in molecular rings of pyrimidines and purines are such parameters: the ring of purine contains 6 atoms and the ring of pyrimidine contains 9 atoms (Figure 7). From the viewpoint of this kind of parameters, C = T = 6, A = G = 9. The ratio 9:6 is equal to the quint 3:2. Thus the symbolic matrices $[A\ C;\ T\ G]^{(n)}$, $[G\ \ C;\ T\ A]^{(n)}$, $[A\ T;\ C\ G]^{(n)}$, $[G\ T;\ A\ C]^{(n)}$ become the threefold quint matrices in the Kronecker power "*n*" in the case of replacement of their symbolic elements by these numbers 9 and 6. The square root of such numeric matrices is connected with the golden matrices obviously. A biological organism is the master on the use of a set of parallel information channels. It is enough to remind about many sensory channels by means of which we obtain sensory information simultaneously: visual, acoustical, tactile, etc. It is probable, that many kinds of genetic matrices are used by organisms in parallel information channels as well.

4 THE GENOMATRICES, MUSICAL HARMONY AND PYTHAGOREAN MUSICAL SCALE

The theme of harmony of living nature is discussed frequently by many authors. The word "harmony" has arisen in Ancient Greece in relation to the Pythagorean musical scale. In the antique theory of music the word "harmony" has found the modern value - the consent of discordant. Seven musical notes carry names familiar to all: do (C), re (D), mi (E), fa (F), sol (G), la (A), si (B). These seven notes are interrelated among themselves by their frequencies not in an accidental manner, but they form the regular uniform ensemble. Really, it is well known that the seven notes of the Pythagorean musical scale from appropriate octaves form the regular sequence of the geometric progression on the base of the quint ratio 3:2 between frequencies of the adjacent members of this sequence (Figures 5). The quint 3:2, which is the ratio between frequencies of the third and the second harmonics of an oscillated string, plays the role of the factor of this geometrical progression. The frequency 293 Hz of the note re (D^1) of the first octave stays in the middle of this frequency series. The ratios of the frequencies of all notes to this frequency of the note re (D^1) form the symmetrical series by signs and sizes of their powers of the quint: from the power "-3" up to the power "+3".

fa (F)	do (C)	sol (G)	re (D^1)	la (A^1)	mi (E^2)	si (B^2)
87	130	196	293	440	660	990
$(3/2)^{-3}$	**$(3/2)^{-2}$**	**$(3/2)^{-1}$**	**$(3/2)^{0}$**	**$(3/2)^{1}$**	**$(3/2)^{2}$**	**$(3/2)^{3}$**

Figure 5 The quint (or the perfect fifth) sequence of the 7 notes of the Pythagorean musical scale. The upper row shows the notes. The second row shows their frequencies. The third row shows the ratios between the frequencies of these notes to the frequency 293 Hz of the note re (D^1). The designation of notes is given on Helmholtz system. Values of frequencies are approximated to integers.

The Kronecker family of the genomatrices $[3\ \ 2;\ 2\ \ 3]^{(n)}$ is connected with the Pythagorean musical scale. Let us consider it more attentively. Each genomatrix of the family $[3\ \ 2;\ 2\ \ 3]^{(n)}$ demonstrates the quint (or the perfect fifth) principle of its structure because they have the quint ratio 3:2 at different levels: between numerical sums in top and bottom quadrants, sub-quadrants, sub-sub-quadrants, etc. including quint ratios between neighbor numbers in them. For example, $[3\ \ 2;\ 2\ \ 3]^{(3)}$ contains 4 numbers – 27, 18, 12, 8 - with the quint ratio between them: 27/18=18/12=12/8=3/2.

Each quint genomatrix $[3\ \ 2;\ 2\ \ 3]^{(n)}$ contains (n+1) kinds of numbers from a geometrical progression, factor of which is equal to the quint 3/2:

$[3\ \ 2;\ 2\ \ 3]^{(1)} \Rightarrow$ 3, 2

$[3\ \ 2;\ 2\ \ 3]^{(2)} \Rightarrow$ 9, 6, 4

$[3\ \ 2;\ 2\ \ 3]^{(3)} \Rightarrow$ 27, 18, 12, 8

…………………………………………

$[3\ \ 2;\ 2\ \ 3]^{(6)} \Rightarrow$ 729, 486, 324, 216, 144, 96, 64

……………………………………………………….

Let us write out these kinds of numbers in columns for each genomatrix $[3\ \ 2;\ 2\ \ 3]^{(n)}$ to arrive at the "genetic" triangle, which is shown on the left part of the expression (1):

$$\begin{array}{cccccc} 3 & 9 & 27 & 81 & 243 & \ldots \\ 2 & 6 & 18 & 54 & 162 & \ldots \\ & 4 & 12 & 36 & 108 & \ldots \\ & & 8 & 24 & 72 & \ldots \\ & & & 16 & 48 & \ldots \\ & & & & 32 & \ldots \end{array} \qquad \begin{array}{cccc} 1 & 3 & 9 & 27 \\ & 2 & & \\ & & 4 & \\ & & & 8 \end{array} \qquad (1)$$

On the right side in the expression (1) the historically famous numeric triangle by Plato is demonstrated. This triangle was utilized by Ancient Greeks to create the Pythagorean musical scale on the basis of its main proportions. One can see the analogy between the "genetic" triangle and the Plato's triangle.

Moreover, as Jay Kappraff (USA) has informed one of the authors of this article in his private letter, this genetic triangle, which was obtained from the matrices of the genetic code, was known many centuries ago: it is identical to the famous triangle, which was published 2000 years ago by Nichomachus of Gerasa in his famous book "Introduction into arithmetic". Nichomachus belonged to the Pythagorean society, and this triangle was famous for centuries as the basis of the Pythagorean theory of musical harmony and aesthetics. In accordance with this triangle, the Parthenon (Kappraff, 2006) and other great architectural objects were created because architecture was interpreted as the non-movement music, and the music was interpreted as the dynamic architecture. Nichomachus of Gerasa was one of the great persons in the theory of musical harmony and aesthetics. The Cambridge library has the ancient picture, where Nichomachus is shown together with other great persons in this field: Pythagoras, Plato and Boeticus (http://www.jcsparks.com/painted/boethius.html). One can find more details about the triangle by Nichomachus of Gerasa in the publications (Kappraff, 2000, 2002). This unexpected connection of times makes additionally probable the adequacy of the presented way of the matrix research of genetic systems and the assumed connection of genetic systems with the Pythagorean musical scale, reflected unconsciously in Nichomachus' triangle.

As we mentioned above, a set of certain kinds of numbers in each genomatrix $[3\ \ 2;\ 2\ \ 3]^{(n)}$ reproduces fragments of the geometrical progressions with the quint factor. Thus sequences of such kinds of numbers can be compared to quint sequences of musical notes from Figure 5. If one confronts the least number from a quint genomatrix with the frequency 87 Hz of the musical note "fa" (F), which possesses the least frequency on Figure 5, then all sequences of such kinds of numbers automatically corresponds to the series of the frequencies of the musical notes: for example, the sequence of numbers 8, 12, 18, 27 of $[3\ \ 2;\ 2\ \ 3]^{(3)}$ is assumed to correspond to the frequency sequence 87, 130, 196, 293 Hz of the notes fa(F) - do(C) - sol(G) - re(D^1). Genomatrix $[3\ \ 2;\ 2\ \ 3]^{(6)}$ contains the sequence of 7 numbers (64, 96, 144, 216, 324, 486, 729), which is assumed to correspond to the whole quint sequence of the frequencies 87, 130, 196, 293, 440, 660, 990 Hz of the 7 notes of Figure 5: fa(F) - do(C) - sol(G) - re(D^1) - la (A^1) - mi (E^2) - si (B^2).

For this reason, we assume that each genomatrix $[3\ \ 2;\ 2\ \ 3]^{(n)}$ can be presented in the form of a matrix $P_{MUSIC}^{(n)}$ of frequencies of notes (or a "music-matrix"). For instance, Figure 6 demonstrates the genomatrix $[3\ \ 2;\ 2\ \ 3]^{(3)}$ of the 64 triplets as a music-matrix $P_{MUSIC}^{(3)}$ of frequencies of appropriate four notes (the general factor 293/27 arises for concordance of numeric values of the note frequencies with numbers 8, 12, 18, 27 of the genomatrix $[3\ \ 2;\ 2\ \ 3]^{(3)}$).

re (D^1)	**sol (G)**	**sol (G)**	**do (C)**	**sol (G)**	**do (C)**	**do (C)**	**fa (F)**
sol (G)	**re (D^1)**	**do (C)**	**sol (G)**	**do (C)**	**sol (G)**	**fa (F)**	**do (C)**
sol (G)	**do (C)**	**re (D^1)**	**sol (G)**	**do (C)**	**fa (F)**	**sol (G)**	**do (C)**
do (C)	**sol (G)**	**sol (G)**	**re (D^1)**	**fa (F)**	**do (C)**	**do (C)**	**sol (G)**
sol (G)	**do (C)**	**do (C)**	**fa (F)**	**re (D^1)**	**sol (G)**	**sol (G)**	**do (C)**
do (C)	**sol (g)**	**fa (F)**	**do (C)**	**sol (G)**	**re (D^1)**	**do (C)**	**sol (G)**
do (C)	**fa (F)**	**sol (G)**	**do (C)**	**sol (G)**	**do (C)**	**re (D^1)**	**sol (G)**
fa (F)	**do (C)**	**do (C)**	**sol (G)**	**do (C)**	**sol (G)**	**sol (G)**	**re (D^1)**

Figure 6 A presentation of the genomatrix $[3\ \ 2;\ 2\ \ 3]^{(3)}*(293/27)$ in the form of the music-matrix $P_{MUSIC}^{(3)}$ of the frequencies of the musical notes (see Figure 5)

The four numbers 8=2*2*2, 12=2*2*3, 18=2*3*3, 27=3*3*3, which are presented in the genomatrix $[3\ \ 2;\ 2\ \ 3]^{(3)}$ on Figure 2, characterize those four kinds of triplets, which differ by their numbers of hydrogen bonds of nitrogenous bases. For instance, number 18=2*3*3 belongs to those triplets, which have one nitrogenous base with 2 hydrogen bond and two bases with 3 hydrogen bonds (the mathematics of genomatrices testifies that products of numbers of hydrogen bonds should be taken into account here but not their sums; it has precedents and the justification in information theories, in particular, in the theory of parallel channels of coding and processing the information). Different sequences of these four numbers, for example 12-8-27-12-8-18-18-…, determine appropriate successions of the musical ratios $(3:2)^0$, $(3:2)^1$, $(3:2)^2$, $(3:2)^3$ (in this example, 3:2 - $(3:2)^3$ – $(2:3)^2$ – (2:3) – $(3:2)^2$ - $(3:2)^0$-…). It is obvious that such succession can be interpreted as a kind of an analogous genetic music for triplets, which is connected with their hydrogen bonds. Each gene and each part of a DNA and RNA have their own genetic "melody of hydrogen bonds" which can be played by means of musical tools.

But the described musical sequence is not the single one in the molecule DNA at all. DNA can be considered as a set of joint sequences, which are very different in their physical-chemical sense: a sequence of nitrogenous bases; a sequence of hydrogen bonds of complementary pairs of these bases; a sequence of triplets; a sequence of rings of nitrogenous bases; a sequence of ensembles of protons in rings of nitrogenous bases, etc. One can note the phenomenological fact that many of these sequences are constructed on quint ratios between quantitative characteristics of their neighboring members, which are typical for the Pythagorean musical scale (it was mentioned above). Correspondingly each of these sequences of ratios can be interpreted as a special kind of genetic musical melody. The whole set of such sequences in DNA can be considered as a polyphonic (coordinated) music ensemble. An investigation of this music ensemble seems to be an important scientific task.

Let us demonstrate a few additional examples of sequences with the musical ratios in DNA. A sequence of triplets in DNA has another kind of genetic music also which is connected with the quantity of protons in molecular rings of nitrogenous bases (Figure 7). The pyrimidines C and T have 40 protons in their rings; the purines A and G have 60 protons in their rings. (Each complementary pair has 100 protons in their rings precisely). The ratio 60:40 is equal to the quint 3:2. Let us present each triplet by the product of the proton numbers 40 and 60 in its rings (as we did above for numbers 2 and 3 of the hydrogen bonds of triplets). Then any triplet has one of four proton numbers: 64000=40*40*40; 96000=40*40*60; 144000=40*60*60; 216000=60*60*60. This proton set of the four numbers differs from the considered set of four numbers 8, 12, 18, 27 of hydrogen bonds in the triplets by the factor 8000 only. In other words, a ratio between any two numbers from this proton set has a quint character again and is equal to one of the values $(3:2)^k$, where $k = \pm 1, 2, 3$. One can note that a sequence of triplets of one DNA-filament has two different sequences with the same typical ratios: one sequence for triplet characteristics of its hydrogen bonds and another sequence for triplet characteristic of protons in triplet rings. These two sequences differ each from other by dispositions of these ratios along DNA-filament, generally speaking (Figure 7). So, any triplet sequence bears on itself two different genetic melodies on these two parameters.

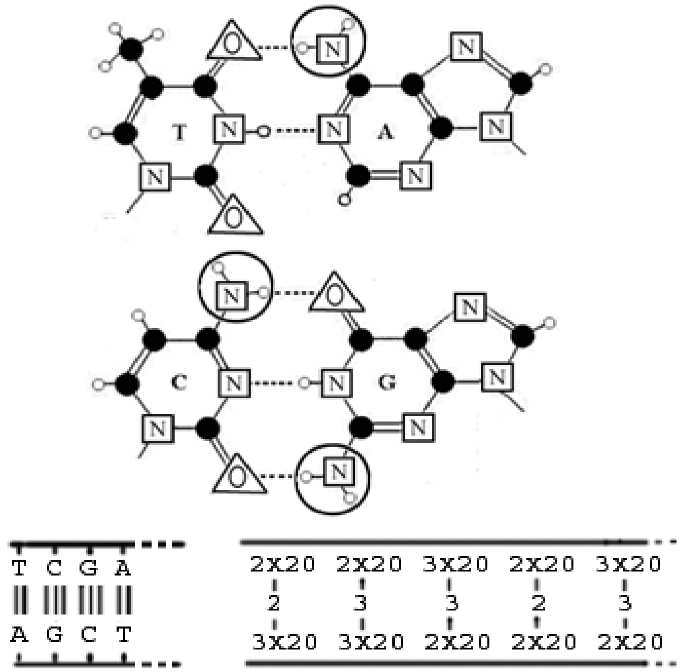

Figure 7 On top: Complementary pairs of four nitrogenous bases in DNA: A - T and C - G. By a dotted line are specified hydrogen bonds in these pairs. Black circles are atoms of carbon, small white circles - hydrogen, circles with the letter N - nitrogen, and circles with the letter O – oxygen. At bottom: the numerical representations of a sequence of complementary pairs of the bases in DNA as a sequence of numbers of hydrogen bonds in the given pairs (the average row made up on basis of numbers 2 and 3) and as a numerical sequence of protons of molecules rings of these nitrogenous bases

Sequential dispositions of musical ratios for these two parameters of triplets (and of nitrogenous bases also) are different on two filaments of DNA, but they are connected in regular manner due to a fact of complementary pairs of bases. Figuratively speaking, two filaments of DNA bear complementary kinds of genetic music on these parameters.

It should be added about an atomic parameter of nitrogenous bases: the quantity of non-hydrogen atoms in molecular rings of the pyrimidines C and T is equal to 6 and the quantity of non-hydrogen atoms in molecular rings of the purines A and G is equal to 9. Their quint ratio 9:6=3:2 can be considered as a basis for “atomic” genetic music of the nitrogenous bases and triplets along DNA. But these kinds of sequences of ratios are identical to sequences of ratios in the case considered above about 40 and 60 protons in rings of the pyrimidines and the purines. For this reason these sequences have nothing new from musical viewpoint though they can have an important meaning in the ensemble of genetic music because they are organized on the higher – atomic - level.

A sequence of numbers of 2 and 3 of hydrogen bonds between complementary nitrogenous bases along DNA (for instance, 3-2-2-3-2-3-…) determines a sequence of ratios between its neighboring - subsequent and previous - members (in the considered example, 2:3 - 2:2 - 3:2 - 2:3 -….). This simple sequence contains ratios $(3/2)^{-1}$, $(3/2)^0$ and $(3/2)^1$ only. From a viewpoint of musical analogy, this sequence determines a special kind of very simple genetic music.

Quantities of molecular rings in the pyrimidines and the purines are characterized by the octave ratio 2:1. This fact gives an additional possibility to consider sequences of nitrogenous bases and triplets in DNA as genetic melodies. But sequences of ratios in these cases contain the octave ratios only and are not so interesting from musical viewpoint though they can play an important role in the whole ensemble of genetic music.

Total quantities of protons in both pairs of nitrogenous bases A-T and C-G are the same and are equal to 136. On this numeric parameter, a sequence of nitrogenous bases has constant ratios 1:1 along DNA.

The full list of different kinds of such genetic music at different parameters and levels of genetic system permits one to reproduce a musical polyphonic party for each gene and for other parts of the genetic system. These musical sequences were created by nature itself. Each gene and each protein have their own genetic music composition (or briefly "genomusic"). The natural music of genes can be reproduced in acoustical diapason not only for aesthetic pleasure but, perhaps, also for medical therapy, for theoretical needs, etc. (applications of genomusic in the field of musical therapy have not been tested by authors). This natural genomusic and its compositions can be connected to deep physiological archetypes, which were introduced into science by the creator of analytic psychology Carl Jung. From the viewpoint of musical harmony in structures of molecular-genetic system, outstanding composers are researchers of harmony in the organization of living substance. According to the famous expression by G. Leibnitz, music is the mysterious arithmetic of the soul, which calculates itself without understanding this action (here one should note the difference in the order of magnitude of the wave-lengths of musical tones and the size of the molecular nitrogenous bases; this fact testifies in favor of informational nature of musical action).

It is well known, that some kinds of music stimulate growth of plants, cure people, etc.

"American Music Therapy Association" unites a few thousands of members, many of them are professional therapists there. One should emphasize that "melodies" of the mentioned genetic music are not formed by any person in a forcible way, but they are defined by natural sequences of parameters in chain genetic molecules (although applications of the genetic music for musical therapy have not been tested else, we may suppose that this kind of music is closer to biological organisms than the former ones). Such genetic melodies are named conditionally as "natural genetic music" to distinguish them from variants of "genetic music", sometimes offered by other authors on the basis of obviously forcible approaches without a sufficient support on molecular features of genetic sequences. The claim is that some authors (see for example http://www.youtube.com/watch?v=tQv5Ho8zsKI) propose their own "genetic music" on the basis of an arbitrary correspondence of the genetic letters or triplets to musical notes without sufficient attention to the musical correspondence of ratios of natural numeric parameters of adjacent genetic elements. Such attempts to create arbitrary "genetic music" are related with the long-standing hypothesis that just the genetic system is the carrier of genetically inherited connection of biological organisms with the phenomenon of music (see for example http://discovermagazine.com/2001/aug/featmusic#.UMyvN0I3tXU).

All physiological systems should be coordinated structurally with the genetic code for their genetic transfer to next generations and for a survival in a course of biological evolution. For this reason we collect examples of harmonious ratios (first of all, the quint 3:2) in structures and functions on different levels of biological systems including the supra-molecular level. For example, the quint ratio 3:2 exists between:

- durations of phases of the activity and the rest in human cardio-cycles (0.6 sec and 0.4 sec correspondingly);
- plasmatic and globular volumes of blood (60% and 40%);
- albumens and globulins of blood (60% and 40%);
- 60S and 40S sub-particles in the composition of ribosomes (from http://vivovoco.rsl.ru/VV/JOURNAL/NATURE/08_03/KISSELEV.HTM).

Now let us consider a well-known algorithm of the construction of the Pythagorean musical scale from a geometrical progression, which factor is equal to the quint. This algorithm, which is useful for the theme of the next paragraph, creates the sequence of the notes do-re-mi-fa-sol-la-si-do on the interval of frequencies {1, 2} of one octave, in which the lowermost note "do" has the conditional frequency of power 1 and the lowermost note of the next octave has the conditional frequency of power 2. This algorithm contains the following steps:

1. Taking the first seven members of such geometrical progression with the quint factor 3/2, which begins from the inverse value of the quint: $(3/2)^{-1}$, $(3/2)^0$, $(3/2)^1$, $(3/2)^2$, $(3/2)^3$, $(3/2)^4$, $(3/2)^5$;

2. Returning into the octave power interval {1, 2} for those members of this sequence, values of which overstep the limits of this interval; this returning is made for these values by means of their multiplication or division with the number 2. As a result of this operation, the new sequence appears (this sequence can be named "the geometrical progression with the returning into the octave "): $2*(3/2)^{-1}$, $(3/2)^0$, $(3/2)^1$, $(3/2)^2/2$, $(3/2)^3/2$, $(3/2)^4/4$, $(3/2)^5/4$;

3. The permutation of these seven members in accordance with their increasing values from 1 up 2 (the number 2 is included in this sequence as the end of the octave): $(3/2)^0$, $(3/2)^2/2$, $(3/2)^4/4$, $2*(3/2)^{-1}$, $(3/2)^1$, $(3/2)^3/2$, $(3/2)^5/4$, 2.

In this last sequence, a ratio of the greater number to the adjacent smaller number refers to as the interval factor. Two kinds of interval factors exist in this sequence only: 9/8, which is named the tone-interval T, and 256/243, which is named the semitone-interval S. One can check that the sequence of interval factors in this case is T-T-S-T-T-T-S. These five tone-intervals and two semitone-intervals cover the octave precisely: $(9/8)^5 * (256/243)^2 = 2$. It is known that the name "semitone-interval" in the Pythagorean musical scale is utilized by convention only because the semitone-interval 256/243= 1.0545… is not equal to the half of the tone-interval, that is the square root from the tone-interval: $(9/8)^{0.5} = 1.0607…$. If one takes not 7, but 6 or 8 members in the initial quint geometrical progression (see the first step of the algorithm), then the same Pythagorean algorithm does not give a binary sequence of interval factors T and S because three kinds of interval factor arise.
The similar algorithm will be used in the next paragraph to construct new mathematical scale on the base of described data about the genetic code and its genomatrices.

5 PENTAGRAM MUSICAL SCALES AND FIBONACCI NUMBERS

Many theorists of music paid attention to the connection of the structure of many musical compositions of prominent composers with the golden section $\varphi = (1+5^{0.5})/2 = 1.618…$ (see, for example, (Lendvai, 1993) and the web-site about the Hungarian composer Bela Bartok and musicologist Erno Lendvai http://mathcs.holycross.edu/~groberts/Courses/Mont2/Handouts/Lectures/Bartok-web.pdf). The results of matrix genetics reveal a new direction of thoughts about a relation between the golden section, Fibonacci numbers and music because structures of a genetic code are also (although in another way) connected with the golden section.

Similarly to a quint genomatrix $[3\ \ 2;\ 2\ \ 3]^{(n)}$, which contains a sequence of (n+1)-kinds of numbers from a geometrical progression with the quint factor 3/2, a corresponding golden genomatrix $\Phi^{(n)}$ contains a sequence of (n+1)-kinds of numbers from a geometric progression, the factor of which is equal to $\varphi^2 = 2.618\ldots.$:

$$\begin{aligned}
&\Phi^{(1)} \Rightarrow \varphi^1,\ \varphi^{-1} \\
&\Phi^{(2)} \Rightarrow \varphi^2,\ \varphi^0,\ \varphi^{-2} \\
&\Phi^{(3)} \Rightarrow \varphi^3,\ \varphi^1,\ \varphi^{-1},\ \varphi^{-3} \qquad (2) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

The previous section demonstrated that the Kronecker family of the quint genomatrices is connected with the Pythagorean musical scale. Now we turn to the Kronecker family of the golden genomatrices and to the geometrical progressions with the factor φ^2. Is it possible to apply the described Pythagorean algorithm to such geometrical progressions with factor φ^2 to arrive at a new musical (or mathematical) scale, where only two interval factors exist by analogy with the Pythagorean musical scale? Investigation of this question seems to be important because such a new scale or scales can be essential for a theory of musical harmony and for the creation of musical compositions with increased physiological activity.

After research of this question the beautiful positive result is obtained: yes, it is possible every time, when we take one of Fibonacci numbers 2, 3, 5, 8, 13 (see the Figure 8) as the first member of such a geometrical progression (the situation becomes more difficult for the higher Fibonacci numbers 21, 34, …). Mathematical scales, which are formed in these cases, possess such quantity of each of their two interval factors, which is equal to Fibonacci numbers as well. Moreover a value of each of these two interval factors is expressed by means of Fibonacci numbers, too.

n	0	1	2	3	4	5	6	7	8	9	10	11	…
F_n	0	1	1	2	3	5	8	13	21	34	55	89	…

Figure 8. The Fibonacci series where $F_{n+1} = F_n + F_{n-1}$

Such interrelated Fibonacci-stage scales, each of which has interval factors of two kinds only and which is based on the geometric progression with the coefficient φ^2, are named "the Fibonacci-stage scales" or "the pentagram scales". Let us consider the example of the 8-stage pentagram scale. We should construct a new mathematical scale of frequencies, which fills up the octave {1, 2}, by means of the Pythagorean algorithm on the base of a geometrical progression with the irrational coefficient φ^2 (instead of the coefficient of the quint 3/2). As a result we should arrive at such a scale, which possesses two kinds of interval factors only by analogy with the Pythagorean musical scale. One can note that the factor $\boldsymbol{\varphi^2}$ = 2.618… exceeds the considered interval of the octave {1, 2}. Therefore it is comfortable to use from the very beginning the twice smaller factor $\varphi^2/2 = p = 1.309$…, the value of which belongs to this octave interval. It is easy to check that the final sequence (3) of the 8-stage pentagram scale does not depend on whether we use the factor φ^2 or the factor $\varphi^2/2$, which are equivalent to each other in the given problem. This factor $p = \varphi^2/2$ has been known in the field of investigations of biological symmetries and morphological invariants for a long time under the name of the golden wurf (Petoukhov, 1981; Petoukhov, He, 2010).

Now let us construct the 8-stage pentagram scale by means of the analogue of the described Pythagorean algorithm, using the factor $p = \varphi^2/2$ in the initial geometric progression (instead of the quint factor 3/2). All three steps of the Pythagorean algorithm are reproduced:

1. Taking the first eight (!) members of such a geometrical progression with the factor $p = \varphi^2/2$, which begins from the inverse value of this factor: $p^{-1}, p^0, p^1, p^2, p^3, p^4, p^5, p^6$;
2. Returning into the octave interval {1, 2} for those members of this sequence, values of which overstep the limits of this interval; this returning is made for these values by means of their multiplication or division with the number 2. As a result of this operation, a new sequence is obtained (this sequence can be named "the geometrical progression with return to the octave "): $2*p^{-1}, p^0, p^1, p^2, p^3/2, p^4/2, p^5/2, p^6/4$;
3. The permutation of these seven members in accordance with their increasing values from 1 up to 2 (the number 2 is included in this sequence as the end of the octave):

$$1,\ p^3/2,\ p^6/4,\ p^1,\ p^4/2,\ 2*p^{-1},\ p^2,\ p^5/2,\ 2 \qquad (3)$$

This final sequence (3) satisfies the initial condition concerning the existence of two kinds of interval factors only. Really, it is easy to check directly that all ratios of adjacent members of this sequence are equal to two values only, which play the role of the interval factors. For this sequence (3), the first kind of intervals is $T = p^3/2$ = 1.1215… and the second kind of intervals is $S = 4*p^{-5}$ = 1.0407… . The sequence of these interval factors is T-T-S-T-S-T-T-S. This sequence fills all the octave in accuracy: $(p^3/2)^5 * (4*p^{-5})^3 = 2$. The quantities of various interval factors are equal to Fibonacci numbers here. Really, the 3 intervals S, 5 intervals T and in total 8 interval factors exist here. It is interesting, that if we take a non-Fibonacci number (for example, 4, 6 or 9) for the first member of the initial geometric progression in the first step of the Pythagorean algorithm, there arise such final sequences, which have more than two kinds of interval factors.

Let us compare the classical 7-stage Pythagorean musical scale with the obtained 8-stage pentagram scale. Figure 9 shows the minimal difference between the sequences (musical scales) of two kinds of intervals inside the octave interval 2 for both scales. The initial and final parts of both sequences coincide completely, and only one additional semitone-interval arises in the middle part of the octave. This additional interval of the second kind S exists because the factor "p" is less than the quint factor.

T	T	S	T		T	T	S
T	T	S	T	S	T	T	S

Figure 9 Sequences of interval factors in the 7-stage Pythagorean scale of C major (the upper row) and in the 8-stage pentagram scale. In each row, the intervals of the first kind are marked by T, and the intervals of the second kind are marked by S (though values of T and S in the upper row differ from values of T and S in the bottom row).

Using the sequence (3) of the intervals, one can construct the sequence of tones (musical notes), which is named the "8-stage pentagram scale of *C* major" by analogy with Pythagorean scale of *C* major (Figure 10). A choice of frequencies for these tones of the first octave is made in such way that this scale contains the frequency 440 Hz, which corresponds to note "la" in the Pythagorean scale and in equal temperament scale and which is used traditionally for tuning in musical instruments. Figure 10 compares the Pythagorean 7-steps scale *C* major and 8-stage pentagram scale for the first octave. Taking into account a minimal difference between the two scales, the majority of the notes of the pentagram scale are named by analogy with the appropriate notes of the Pythagorean scale but with the letter "m" in the end (for instance, "rem" instead "re"). The additional fifth note is named "pim".

260.7 DO_1	293.3 RE	330.0 MI	347.6 FA		391.1 SOL	440 LA	495.0 SI	521.5 DO_2
256.8 DOM_1	288.0 REM	323.0 MIM	336.1 FAM	376.8 PIM	392.3 SOLM	440 LAM	493.5 SIM	513.6 DOM_2

Figure 10 The upper row demonstrates the frequencies of the tones in the 7-stage Pythagorean scale of C major in the first octave. The bottom row demonstrates the

frequencies of the tones in the 8-stage pentagram scale of C major in the similar octave. Numbers mean frequencies in Hz. The names of the notes are given.

This pentagram scale, which was constructed in connection with parameters of the genetic code, possesses many analogies with the Pythagorean musical code by their internal symmetries and proportions. Its main difference from the Pythagorean scale is connected with irrational values of its interval factors. Irrational factors are used also in the modern equal-temperament scale. According to some data, Ancient Chinese knew about the equal-temperament scale, but neglected it preferring the Pythagorean scale, in which they saw cosmic and biological importance.

The history of attempts of creation of new musical scales includes names of many prominent scientists: J. Kepler, R. Descartes, G. Leibnitz, L. Euler, etc. But these authors had no possibility to use the data about the genetic code in their attempts. The data about the genetic code allow one to create new musical scales.

By analogy with the 8-stage pentagram scales, other Fibonacci-stage scales can be constructed. Figure 11 shows both kinds of interval factors T and S in the pentagram scales with different Fibonacci stages (see more details in (Petoukhov, 2008)).

№	Scales	Value of T_K	Value of S_K	Sequence of T_K и S_K in the scale
$K = 0$	1	2	-	T_0
$K = 1$ $(n=3)$	2	2^1*p^{-1} = 1,5279…	2^0*p^1 = 1,3090…	T_1 –S_1
$K = 2$ $(n=4)$	3	2^0*p^1 = 1,3090…	2^1*p^{-2} = 1,1672…	T_2– S_2 – T_2
$K = 3$ $(n=5)$	5	$2*p^{-2}$ = 1,1672…	$2^{-1}*p^3$ = 1,1215…	S_3–T_3–T_3 –S_3 –T_3
$K = 4$ $(n=6)$	8	$2^{-1}*p^3$ = 1,1215…	2^2*p^{-5} = 1,0407…	T_4–T_4–S_4–T_4–S_4–T_4–T_4–S_4
$K = 5$ $(n=7)$	13	2^2*p^{-5} = 1,0407…	$2^{-3}*p^8$ = 1,0776…	S_5-T_5-S_5-T_5-T_5-S_5-T_5-T_5-S_5-T_5-S_5-T_5-T_5
$K = 6$ $(n=8)$	21	$2^{-3}*p^8$ = 1,0776…	2^5*p^{-13} = 0,9657…	T_6-T_6-S_6-T_6-T_6-S_6-T_6-S_6-T_6-T_6-S_6-T_6-S_6-T_6-T_6-S_6-T_6-T_6-S_6-T_6-S_6
$K = 7$ $(n=9)$	34	2^5*p^{-13} = 0,9658…	$2^{-8}*p^{21}$ = 1,1159…	S_7-T_7-S_7-T_7-T_7-S_7-T_7-S_7-T_7-T_7-S_7-T_7-T_7-S_7-T_7-S_7-T_7-T_7-S_7-T_7-T_7-S_7-T_7-S_7-T_7-T_7-S_7-T_7-S_7-T_7-T_7-S_7-T_7-T_7

$K = 8$ (n=10)	55	$2^{-8}*p^{21}$ = 1,1159…	$2^{13}*p^{-34}$ = 0,8655…	T_8-T_8-S_8-T_8-T_8-S_8-T_8-S_8-T_8-T_8-S_8-T_8-T_8-S_8-T8-S8-T_8-T_8-S_8-T_8-S_8-T_8-T_8-S_8-T_8-T_8-S_8-T_8-S_8-T_8-T_8-S_8-T_8-S_8-T_8-T_8-S_8-T_8-T_8-S_8-T_8-S_8-T_8-T_8-S_8-T_8-T_8-S_8-T_8-S_8-T_8-T_8-S_8-T_8-S_8

Figure 11 The values and the order of both kinds of intervals T_K and S_K in the first Fibonacci-stage pentagram scales. "K" means a serial number of pentagram scales; "n" is a serial number of Fibonacci values from Figure 8.

Each of pentagram scales (Figure 11) contains a Fibonacci quantity of each of interval factors T_K and S_K: the interval T_K is repeated F_{n-1} times and the interval S_K is repeated F_{n-2} times. In total, the number of repetitions of these intervals T_K+S_K is equal to F_n, and they always exhaust the octave interval 2 exactly:

$$(T_K\text{^}F_{n-1})*(S_K\text{^}F_{n-2}) = 2, \qquad (4)$$

where the symbol «^» means exponentiation.

In addition, values of each of T_K and S_K are also expressed via Fibonacci numbers simply.
One can see from the table on Figure 11 that the recurrent relations exist for the system of the pentagram scales:

$$T_{K+2} = T_K/T_{K+1}; \quad S_K = T_{K+1} \quad (K = 0, 1, 2,..; T_0 = 2, T_1 = 2*p^{-1}) \qquad (5)$$

These relations lead to a recurrent algorithm (6) for calculating interval factors in the pentagram scales. This new algorithm can be considered as an alternative variant in relation to the Pythagorean algorithm described above. On the base of values T_1 and T_2, this new algorithm allow calculating the values of the interval factors T_K for K = 3, 4, 5, 6,…, which correspond to 3-, 5-, 8-, 13-, 21- and higher order of the Fibonacci-stage scales without using the Pythagorean algorithm. Really the recurrent relations (5) generate relations (6) for values T_K as functions T_1 and T_2:

$$T_K=[(T_1\text{^}F_{K-2})/(T_2\text{^}F_{K-1})]\text{^}(-1)^{K+1} \qquad (6)$$

where the symbol «^» means exponentiation; F_{K-1} and F_K are Fibonacci numbers.

Due to the expression $T_{n+2}=T_n/T_{n+1}$ (5) this family of Fibonacci-stage scales is connected with Pascal's triangle and the coefficients of the binomial expansion by Newton because $T_0^1 = T_1^1 * T_2^1 = T_2^1 * T_3^2 * T_4^1 = T_3^1 * T_4^3 * T_5^3 * T_6^1 = T_4^1 * T_5^4 * T_6^6 * T_7^4 * T_8^1$ = … . The exponents in these products coincide with the binomial coefficients.

Another algorithm exists to determine the order of T_K and S_K in each of the pentagram scales on the base of knowledge about their order in the first pentagram scales: T_0 and T_1-S_1. This algorithm is connected with the classical task by Fibonacci about rabbits' reproduction. The algorithm is based on the fact that under transition from the pentagram scale K to the next pentagram scale K+1, each interval T_K is replaced by two intervals T_{K+1} and S_{K+1}, and each interval S_K is replaced by interval T_{K+1}. It should be noted that under transition from the scale with odd numeration K to the scale with even numeration (for example, from K=3 to K=4) the interval T_K is replaced by T_{K+1} and S_{K+1} (the order T_{K+1} and S_{K+1} is essential here). In contrary, under transition from the scale with even numeration K to the scale with odd numeration (for example, from K=4 to K=5) the interval T_K is replaced by S_{K+1} and T_{K+1} (the reverse order).

Let us explain this with an example of the sequence S_3-T_3-T_3-S_3-T_3, pointing in brackets for each of S_3 and T_3 their algorithmic transformation into T_4 and S_4 under transition from the pentagram scale with K=3 to the next scale with K = 4: $S_3(T_4)$–$T_3(T_4$-$S_4)$–$T_3(T_4$-$S_4)$–$S_3(T_4)$–$T_3(T_4$-$S_4)$. Paying attention only to the sequence of T_4 and S_4 inside brackets, we get the familiar sequence T_4–T_4–S_4–T_4–S_4–T_4–T_4–S_4 for the pentagram scale with K=4 on Figure 11.

Figure 12 shows the tree of reproduction of interval factors T_K and S_K, which is constructed on the base of this algorithm and which corresponds to sequences of T_K and S_K in the table of pentagram scales on Figure 11.

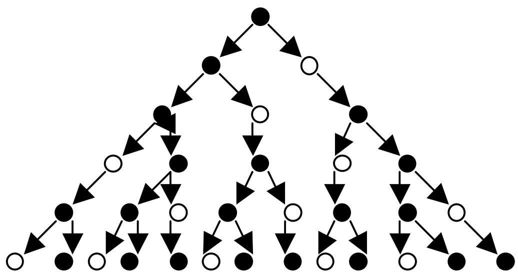

Figure 12 The tree of reproduction of intervals T_K (black circles) and S_K (white circles) for the ensemble of the Fibonacci-stage pentagram scales from the table on Figure 11.

But the similar tree in which each of two elements is a repeated Fibonacci number at each level, is known since before the Renaissance. It appeared in the "biological" task by Fibonacci. This task speaks about breeding rabbits, a couple of which give birth every month a new pair, but give birth to rabbits only from the second month of its birth. The history of the task and applications of Fibonacci numbers in many fields of contemporary science are described in the book (Vorobyev, 2003).

Along with the similarities between trees in the Fibonacci's classical task and in our musical "task of the octave", which is associated with expression (4), the following mathematical differences exist between these tasks and between their trees:

- Our task of the octave analyzes not only the number of T_K and S_K, but also the values of each of the T_K and S_K, which is expressed through the Fibonacci numbers. Fibonacci's classical task doesn't consider parameters of each rabbit (e.g., weight or size), and all rabbits are different from each other only on the basis of sexual maturity.
- The octave task determines the order of T_K and S_K for each level of the tree of the pentagram scales. Fibonacci's task doesn't consider the order of two kinds of rabbits (which reached or not reached their sexual maturity) at each level of the Fibonacci tree.
- Octaves are not considered at all in the frame of the Fibonacci task.

Taking these facts into account, our "task of the octave" can be represented as complication or generalization of the classical Fibonacci task.

On the base of the table in Figure 11, one could construct systems of sound frequencies for the pentagram scales corresponding to different Fibonacci stages. In this case a new interesting result appears: all the musical frequencies of any pentagram scale are repeated in all higher pentagram scales, which have more numbers of steps. In other words, a fractal-like principle exists which provides incorporations of the set of sound frequencies of lower pentagram scale into the set of frequencies of higher pentagram scales. Each subsequent pentagram scale contains information about musical frequencies of all previous "generations" of pentagram scales.

The Moscow State Conservatory by P.I. Tchaikovsky has recently created a special “Center for interdisciplinary research of musical creativity” headed by one of the authors of the article. One of the main tasks of this center is to study genetic musical scales described in this article. This study is being conducted now from different viewpoints including new opportunities for composers. The study revealed that the pentagram musical scales possess beautiful and rich harmony for sound perception and they can be used to compose music based on them. The system of the musical pentagram scales has much more possibilities to produce harmonic sounds in comparison with the equal tempered scale, which is widely used now. The authors of the article have created a few musical instruments and special software on the computer language Python to produce appropriate musical products, which are used in this study. A group of specialists from different fields of science, medicine and culture participate also in these works. In addition, theoretical researches in this field are conducted in the international institute “Symmetrion” (Budapest, Hungary). Initial results of the wide study testify into a favor of great perspectives of this direction for science and culture.

In our opinion, the aesthetic aspects of genetic music are connected not with a mechanical resonance of molecular structures under influence of sound waves but with informational aspects, which provide an effect of (not yet identified way of) recognition of a kindred language under during listening genetic music. This effect of recognition can be provided by biological algorithms of signal processing inside organisms. For example, in the case of pentagram music from the outside world, our organism can recognize those ratios, on which our genetic system and the whole inherited physiology are built, and the organism responds positively to this manifestation of a structural kinship of the outside world with its own genetic physiology. This positive reaction can be compared with mutual understanding between two persons when they begin to talk in the same language (if they talk in different languages, mutual understanding and interactivity don't arise though these persons can speak more and more loudly and energetically). Music is not limited to the relationship of sounds emitted by a system of stretched strings, which were studied by Pythagoras. The purpose of music is to call the emotion, associations and living pictures from bio-informational memory. This may be made effectively not only by sounds from classical musical systems of stretched strings but also (and more effectively?) by other sets of sounds, which are structured and toned on the base of inherited algorithms of biological processing of genetic information. No wonder the sense of musical harmony is innate.

6 SOME CONCLUDING REMARKS

The facts described in this article about relations of the genetic systems with musical harmony are essential for the problem of genetic bases of aesthetics and inborn feeling of harmony. According to the words of the famous physicist Richard Feynman about feeling of musical harmony, "*we may question whether we are any better off than*

Pythagoras in understanding why [stressed] only certain sounds are pleasant to our ear. The general theory of aesthetics is probably no further advanced now than in the time of Pythagoras" (Feynman, Leighton, & Sands, 1963, Chapter 50).

A cultural direction of "genetic art" (or briefly "genoart") can be developed additionally due to these data of matrix genetics. Genoart has many patterns, which are revealed by matrix genetics, and can be used to create new works of art, of designs and architectural and musical compositions. For example, the quint genomatrices can be presented in a form of color mosaics if matrix numbers are replaced by colors. It is possible to see regular complication of color mosaics along the family of the genomatrices with an increase of their Kronecker powers. The discovery of the connection of the genetic code with the golden section shows the molecular-genetic base of many known facts about aesthetic meanings of the golden section. Specifically the described facts give new materials for the question about architectural canons, where the golden section is used for a long time; for example, the famous modulor by Le Corbusier (1948, 1953, http://en.wikipedia.org/wiki/Le_Corbusier) is based on the golden section. The pentagram Fibonacci-stage scales can be additionally utilized for architectural proportions (in the role of "pentagram modulor").

There is no doubt that applications of numeric genetic matrices for investigations of the various ensembles of parameters of the genetic system can give many unexpected and useful results in the future as well. This direction of theoretical researches will be developed in parallel with developing matrix application in many other branches of science. The matrix-genetic approach to phenomena of the golden section in genetic systems and aesthetics can be developed in many theoretical ways and can give new interesting mathematical models.

According to the described materials, each gene, each DNA, each protein can be characterized by its own "musical ensemble". Sequences of appropriate musical intervals from such genetic melodies can be reproduced in a form of sequences of sounds, colors ("color music"), electrical stimulus, and impulses of laser beams, etc. for different needs (though own frequencies of these physical matters are very different). Whether such "natural genetic music" (or compositions on its basis) possesses a special physiological effectiveness for the treatment of people and animals, stimulation of growth of plants and microorganisms, and so forth? Only future experiments can give the answer more precisely. It seems that a creation of a computer bank of genetic music is useful for theoretical and practical needs. One can add here that the creator of analytic psychology Carl Jung, studying archetypes of human consciousness, has created the medical method of amplification. This method is based on an active intercourse of his patients with these archetypes including famous tables of Ancient Chinese "I Ching", which are connected with the genetic matrices (Petoukhov, 2005, 2008; Petoukhov, He, 2010; Tusa, 1994). Many composers declared a mysterious connection of music with the golden section and Fibonacci numbers early. In our opinion, this connection has based on the musical scale tuned on the described scale, which was constructed on the analogy of the mathematical sequences discovered in the algebraic structure of the genetic coding. The described facts are related to a problem of genetic bases of aesthetics and an inborn feeling of harmony.

Investigations of numeric genetic matrices are an effective scientific instrument to analyze multi-component and multi-parametric ensembles of the molecular-genetic systems. The obtained results give a new vision of connections of genetic systems with well-known mathematical objects and theories from other branches of science and culture. Owing to the results of matrix genetics new opportunities arise to demonstrate the close connection between science and culture. One of them is a problem of multi-dimensional spaces including multi-dimensional musical spaces which need appropriate algebraic formalisms for their analysis (Kappraff, Petoukhov, 2009; Koblyakov, 1995, 2000a,b).

One should note that our attempt to create the mathematical scale of the golden section, where the factor of the geometrical progression is equal to the golden section φ (but not to the φ^2), has led to a scale, which differs from the Pythagorean musical scale cardinally and which has been considered not so interesting from the musical viewpoint. Furthermore such scales of the golden section had no evident connection with Fibonacci numbers in its interval factors.

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